ĀRYABHAṬĪYA

OF

ĀRYABHAṬA

Critically edited
with Introduction, English Translation,
Notes, Comments and Indexes

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INDIAN NATIONAL SCIENCE ACADEMY
NEW DELHI

Published for

THE NATIONAL COMMISSION FOR THE COMPILATION OF HISTORY OF SCIENCES IN INDIA

by

The Indian National Science Academy
Bahadur Shah Zafar Marg, New Delhi—1

Rs. 21.50 (in India)
$ 7.00 ; £ 2.75 (outside India)

Printed in India

At the Vishveshvaranand Vedic Research Institute Press
Sadhu Ashram, Hoshiarpur (Pb.)

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LIST OF ABBREVIATIONS

1. BOOKS

Ā	*Āryabhaṭīya* of Āryabhaṭa I
ĀSi	*Āryabhaṭa-siddhānta*
BBi	Bhāskara II's *Bījagaṇita*
BM	*Bakhshali Manuscript*
BṛJā	*Bṛhat-jātaka* of Varāhamihira
BṛSaṁ	*Bṛhat-saṁhitā* of Varāhamihira
BrSpSi	*Brāhma-sphuṭa-siddhānta* of Brahmagupta
GCN	*Graha-cāra-nibandhana* of Haridatta
GSS	*Gaṇita-sāra-saṅgraha* of Mahāvīra
GK	*Gaṇita-Kaumudī* of Nārāyaṇa
GT	*Gaṇita-tilaka* of Śrīpati
KK	*Khaṇḍa-khādyaka* of Brahmagupta
KR	*Karaṇa-ratna* of Deva
L	*Līlāvatī* of Bhāskara II
L(ASS)	*Līlāvatī* (Ānandāśrama Sanskrit Series)
LBh	*Laghu-Bhāskarīya* of Bhāskara I
LMā	*Laghu-mānasa* of Mañjula (Muñjāla)
MBh	*Mahā-Bhāskarīya* of Bhāskara I
MSi	*Mahā-siddhānta* of Āryabhaṭa II
NBi	Nārāyaṇa's *Bījagaṇita*
PG	*Pāṭīgaṇita of* Śrīdhara
PSi	*Pañca-siddhāntikā* of Varāhamihira
PuSi	*Puliśa-siddhānta*
RoSi	*Romaka-siddhānta*
ŚiDVṛ	*Śiṣya-dhī-vṛddhida* of Lalla
SiŚe	*Siddhānta-śekhara* of Śrīpati
SiŚi	*Siddhānta-śiromaṇi* of Bhāskara II
SiTV	*Siddhānta-tattva-viveka* of Kamalākara
SMT	*Sumati-mahā-tantra* of Sumati
SuSi	*Sundara-siddhānta* of Jñānarāja
SūSi	*Sūrya-siddhānta*
Triś	*Triśatikā* of Śrīdhara
VSi	*Vaṭeśvara-siddhānta*

2. PERIODICALS ETC.

ABORI	*Annals of the Bhandarkar Oriental Research Institute*
AMM	*American Mathematical Monthly*

An. SS	*Ānandāśrama Sanskrit Series*
AR	*Asiatick Researches*
BCMS	*Bulletin of the Calcutta Mathematical Society*
BM	*Bibliotheca Mathematica*
BNISI	*Bulletin of the National Institute of Sciences of India*
Ep. Ind.	*Epigraphia Indica*
IC	*Indian Culture*
IJHS	*Indian Journal of History of Science*
IHQ	*Indian Historical Quarterly*
INSA	Indian National Science Academy
JA	*Journal Asiatique*
JAOS	*Journal of the American Oriental Society*
JBBRAS	*Journal of the Bombay Branch of the Royal Asiatic Society*
JASB	*Journal of the (Royal) Asiatic Society of Bengal*
JASGBI	*Journal of the Asiatic Society of Great Britain and Ireland*
JBORS	*Journal of the Bihar and Orissa Research Society*
JBRS	*Journal of the Bihar Research Society*
JDL/CV	*Journal of the Department of Letters of the Calcutta University*
JIBS	*Journal of Indian and Buddhist Studies* (Tokyo)
JIMS	*Journal of the Indian Mathematical Society*
JORM	*Journal of Oriental Research, Madras*
Math. Edu.	*Mathematics Education*
QJMS	*Quarterly Journal of the Mythic Society*
TSS	*Trivandrum Sanskrit Series*
ZDMG	*Zeitschrift für Deutsche Morgenlandischen Gesellschaften*

3. COMMENTATORS

Bh.	Bhāskara I	Ra.	Raghunātha-rāja
Br.	Brahmagupta	Śa.	Śaṅkaranārāyaṇa
Go.	Govinda-svāmī	So.	Someśvara
Kṛ.	Kṛṣṇadāsa	Sū.	Sūryadeva
Nī.	Nīlakaṇṭha	Ud.	Udayadivākara
Pa.	Parameśvara	Ya.	Yallaya
Pṛ.	Pṛthūdaka		

ROMAN TRANSLITERATION OF DEVANAGARI

VOWELS

Short :	अ	इ	उ	ऋ	लृ (and ॡ)		
	a	i	u	ṛ	ḷ		
Long :	आ	ई	ऊ	ए	ओ	ऐ	औ
	ā	ī	ū	e	o	ai	au

Anusvāra : ं = ṁ

Visarga : ः = ḥ

Non-aspirant : ऽ = ʃ

CONSONANTS

Classified :	क्	ख्	ग्	घ्	ङ्			
	k	kh	g	gh	ṅ			
	च्	छ्	ज्	झ्	ञ्			
	c	ch	j	jh	ñ			
	ट्	ठ्	ड्	ढ्	ण्			
	ṭ	ṭh	ḍ	ḍh	ṇ			
	त्	थ्	द्	ध्	न्			
	t	th	d	dh	n			
	प्	फ्	ब्	भ्	म्			
	p	ph	b	bh	m			
Un-classed :	य्	र्	ल्	व्	श्	ष्	स्	ह्
	y	r	l	v	ś	ṣ	s	h
Compound :	क्ष्	त्र्	ज्ञ्					
	kṣ	tr	jñ					

INTRODUCTION

The present volume, which forms Part I of our edition of the *Āryabhaṭīya*, contains a critically edited text of the *Āryabhaṭīya* and its English translation along with explanatory and critical notes and comments.

1. ĀRYABHAṬA—THE AUTHOR

The *Āryabhaṭīya* is a composition of Āryabhaṭa. The author mentions his name at two places in the *Āryabhaṭīya*, first in the opening stanza of the first chapter (*viz.*, *Gītikā-pāda*) and then in the opening stanza of the second chapter (*viz.*, *Gaṇita-pāda*). In the concluding stanza, he calls the work *Āryabhaṭīya* ('A composition of Āryabhaṭa') after his own name.

This Āryabhaṭa is a different person from his namesake of the tenth century A.D., the author of the *Mahā-siddhānta*. To distinguish between the two, the author of the *Āryabhaṭīya* is called Āryabhaṭa I, and the author of the *Mahā-siddhānta* is called Āryabhaṭa II.

It is Āryabhaṭa I, author of the *Āryabhaṭīya*, after whose name the first Indian satellite was designated 'Āryabhaṭa' and put into orbit on April 19, 1975 and whose 1500th birth anniversary is being celebrated now.

2. HIS PLACE

2.1. Kusumapura

Āryabhaṭa I does not expressly state the place to which he belonged, but he mentions Kusumapura and there are reasons to believe that he lived at Kusumapura and wrote his *Āryabhaṭīya* there. In stanza 1 of chapter ii of the *Āryabhaṭīya*, he writes:

> "Āryabhaṭa sets forth here the knowledge honoured at Kusumapura."[1]

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1. आर्यभटस्त्विह निगदति कुसुमपुरेऽभ्यर्चितं ज्ञानम् । (*Ā*, ii. 1)

The commentator Parameśvara (A.D. 1431) interprets this statement as meaning :

> "Āryabhaṭa sets forth in this country called Kusumapura, the knowledge honoured at Kusumapura."

The commentator Raghunātha-rāja (A.D. 1597), too, interprets it in a similar way :

> "Āryabhaṭa, while living at Kusumapura, sets forth the knowledge honoured at Kusumapura."

That Āryabhaṭa I belonged to Kusumapura is substantiated by the following stanza which is quoted in connection with Āryabhaṭa I :

> "When the methods of the five *Siddhāntas* began to yield results conflicting with the observed phenomena such as the settings of the planets and the eclipses, etc., there appeared in the Kali age at Kusumapurī Sūrya himself in the guise of Āryabhaṭa, the *Kulapa* well versed in astronomy."[1]

The Persian scholar Al-Bīrūnī (A.D. 973-1048), too, has, on occasions more than one, called him 'Āryabhaṭa of Kusumapura'.[2]

Bhāskara I (A.D. 629), the earliest commentator of the *Āryabhaṭīya*, identifies Kusumapura with Pāṭaliputra in ancient Magadha, and 'the knowledge honoured at Kusumapura' with the teachings of the *Svāyambhuva-* or *Brahma-siddhānta*. He also informs us that at Magadha the year commenced on the first *tithi* of the dark half of the month Śrāvaṇa and ended on the fifteenth *tithi* of the light half of the month Āṣāḍha. From the writings of the early Jaina scholars who belonged to Kusumapura we know that the astronomers of Pāṭaliputra in Magadha were the followers of the Brahma school. We also know that in Magadha, since A.D. 593 down to the present day, the year, which is known as '*Sāla*' there, is taken to commence from the first *tithi* of the dark half of the month Śrāvaṇa.

Hence we can conclude without any shadow of doubt that Āryabhaṭa I flourished at Kusumapura or Pāṭaliputra in ancient Magadha, or modern Patna (long. 25°·37 N., lat. 85°·13 E.) in Bihar State.

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1. सिद्धान्तपञ्चकविधावपि दृग्विरुद्धमौढ्योपरागमुखखेचरचारक्लृप्तौ ।
सूर्यः स्वयं कुसुमपुर्यभवत् कलौ तु भूगोलवित् कुलप आर्यभटाभिधानः ॥

2. See for example, *Al-Bīrūnī's India*, translated by E.C. Sachau, Vol. I, London (1910), pp. 176, 246, 330 and 370.

Repeated homage to Brahmā[1] (the promulgator of the *Svāyambhuva-siddhānta*) and acknowledgement to 'the grace of Brahmā'[2] in the *Āryabhaṭīya*, also point to the same conclusion.

2.2. Aśmaka

Bhāskara I (629 A.D.), the commentator of the *Āryabhaṭīya*, refers to Āryabhaṭa I as Aśmaka, his *Āryabhaṭīya* by the names *Āśmaka-tantra* and *Āśmakīya*, and his followers by the designation *Āśmakīyāḥ* at several places in his writings in more than one context.

The use of the above-mentioned words shows that Āryabhaṭa I was an Aśmaka, *i.e.*, his original homeland was Aśmaka. According to the commentator Nīlakaṇṭha (1500 A.D.), he was born in the Aśmaka Janapada.[3] (For Aśmaka, see vol. II, introduction, pp. xxvii-xxviii).

It seems that Āryabhaṭa I was an Aśmaka who lived at Pāṭaliputra (modern Patna) in Magadha (modern Bihar) and wrote his *Āryabhaṭīya* there. Magadha in ancient times was a great centre of learning. The famous University of Nālandā was situated in that state in the modern district of Patna. There was a special provision for the study of astronomy in this University. According to D.G. Apte,[4] an astronomical observatory was a special feature of this University. In a passage quoted above, Āryabhaṭa I has been designated as *Kulapa* (=*Kulapati* or Head of a University). It is quite likely that he was a *Kulapati* of the University of Nālandā which was in a flourishing state in the fifth and sixth centuries A.D. when Āryabhaṭa I lived.

3. HIS TIME

The year of birth of Āryabhaṭa I is known to us with precision. There is a verse in the *Āryabhaṭīya* which runs as follows : "When sixty times sixty years and three quarter-*yugas* had elapsed (of the

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1. *Ā*, i. 1 ; *Ā*, ii. 1.
2. *Ā*, iv. 49.
3. See, opening lines of Nīlakaṇṭha's comm. on *Gaṇitapāda*.
4. See *Universities in Ancient India*, p. 30.

current *yuga*), twenty-three years had then passed since my birth."[1] This shows that in the Kali year 3600 (elapsed), Āryabhaṭa I was twenty-three years of age. Since the Kali year 3600 (elapsed) corresponds to A.D. 499, it follows that Āryabhaṭa I was born in the year A.D. 476. The Gupta king Buddhagupta reigned at Pāṭaliputra from A.D. 476 to A.D. 496. This shows that Āryabhaṭa I was born in the same year in which Buddhagupta took over the reigns of government at Pāṭaliputra.

To be more precise, 3600 years of the Kali era came to an end on Sunday, March 21, A.D. 499, at mean noon at Laṅkā or Ujjayinī, at the time of Mean Sun's entrance into the sign Aries (*madhyama-meṣa-saṅkrānti*) (See the table given below). So, the time of birth of Āryabhaṭa I may be fixed at *Meṣa-saṅkrānti* on March 21, A. D. 476. Since at the end of the Kali year 3600 the precession of the equinoxes amounted to zero (see the next paragraph), the amount of the precession of the equinoxes 23 years before the time of Āryabhaṭa's birth was negligible, Hence his birth may be taken to have occurred at *nirayaṇa-meṣa-saṅkrānti* or at *sāyana-meṣa-saṅkrānti*. The Bihar Research Society, Patna, celebrates the birth anniversary of Āryabhaṭa on April 13, the day on which the Sun now enters into the *nirayaṇa* sign Aries (*i. e.*, on the *nirayaṇa meṣa-saṅkrānti* day).

Mean positions of the Planets[2]
at Kali 3600 elapsed, *i.e.*, on Sunday, March 21, A.D. 499, mean noon at Ujjayinī.

Planet	*Āryabhaṭīya*	*Āryabhaṭa-siddhānta*	Ptolemy	Moderns
Sun	0° 0′ 0″	0° 0′ 0″	357° 8′ 16″	359° 42′ 5″
Moon	280° 48′ 0″	280° 48′ 0″	278° 24′ 58″	280° 24′ 52″
Moon's apogee	35° 42′ 0″	35° 42′ 0″	32° 43′ 42″	35° 24′ 38″
Moon's asc. node	352° 12′ 0″	352° 12′ 0″	349° 25′ 33″	352° 2′ 26″
Mars	7° 12′ 0″	7° 12′ 0″	4° 20′ 12″	6° 52′ 45″
Mercury	186° 00′ 0″	180° 0′ 0″	178° 0′ 27″	183° 9′ 51″
Jupiter	187° 12′ 0″	186° 0′ 0″	185° 20′ 55″	187° 10′ 47″
Venus	356° 24′ 0″	356° 24′ 0″	351° 4′ 15″	356° 7′ 51″
Saturn	49° 12′ 0″	49° 12′ 0″	45° 55′ 39″	48° 21′ 13″

1. षष्ट्यब्दानां षष्टिर्यदा व्यतीतास्त्रयश्च युगपादाः ।
   त्र्यधिका विंशतिरब्दास्तदेह मम जन्मनोऽतीताः ॥ (*Ā*, iii. 10)
2. Taken from *Siddhānta-śekhara* of Śrīpati, Part II, edited by

It may be asked : What consideration prompted Āryabhaṭa I to mention the end of the Kali year 3600 which happened to occur on Sunday, March 21, A.D. 499, at mean noon at Ujjayinī ? Or, does it denote the time of composition of the *Āryabhaṭīya* ? According to the commentators of the *Āryabhaṭīya*, the object of specifying the end of the Kali year 3600 was to show that at that time the precession of the equinoxes amounted to zero and the mean positions of the planets obtained from the astronomical parameters given in the *Gītikā-pāda* did not require any correction. The commentator Sūryadeva (*b*. A.D. 1191), Parameśvara (A.D. 1431) and Nīlakaṇṭha (A.D. 1500), however, are of opinion that this was also the time of composition of the *Āryabhaṭīya*. K. Sāmbaśiva Śāstrī, W.E. Clark and Baladeva Misra, too, hold the same opinion. P.C. Sengupta once entertained this view but later discarded it.

The Kerala astronomer Haridatta (also called Haradatta) (*c*. A.D. 683), the alleged author of the so-called Śakābda correction (with epoch at Śaka 444), has, as remarked by the commentator Nīlakaṇṭha[1] (rather in surprise), interpreted the above-mentioned verse of the *Āryabhaṭīya* (*viz*. iii. 10) in a different way : "When sixty times sixty years and three quarter-*yugas* had elapsed (of the current *yuga*), twenty-three years of my age have passed since then." No commentator of the *Āryabhaṭīya*, not even of Kerala, has interpreted the above passage in this way. T. S. Kuppanna Sastri has called it a wrong interpretation.[2] Another Kerala astronomer (probably Jyeṣṭhadeva), author of the *Dṛkkaraṇa* (A.D. 1603), an astronomical manual in Malayaḷam, has actually stated that Āryabhaṭa I was born in A.D. 499 and that Āryabhaṭa I wrote the *Āryabhaṭīya* twenty-three

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Babuaji Misra, Calcutta, 1947, introduction by P.C. Sengupta and N. C. Lahiri, p. xii.

1. See his commentary on *Ā*, iv. 48, p. 150 of the printed edition, *Trivandrum Sanskrit Series*, No. 185.

2. See *Mahābhāskarīyam*, edited by T.S. Kuppanna Śāstrī, introduction, p. xvi.

years later, in 522 A.D.[1] This, according to T.S. Kuppanna Śāstrī, is a mistaken impression.[2]

It must be noted that the translation of the verse in question as given earlier (on p. xix-xx) is in agreement with the interpretation of the commentators. This is also in conformity with what, according to Bhāskara I, Āryabhaṭa I himself told his pupils while teaching the subject.

However, the duality of interpretation of the above verse has given rise to two epochs (called *bhaṭābda*, 'the year of Āryabhaṭa') associated with Āryabhaṭa I, *viz.* Śaka 421 (=A.D. 499) and Śaka 444 (=A.D. 522), and to two *bīja* corrections, one taking the beginning of Śaka 421 and the other the beginning of Śaka 444 as the zero point.

4. HIS PUPILS

No more information regarding the life of Āryabhaṭa I is now available to us. The *Āryabhaṭīya* does not throw light on such aspects as his parentage, his educational careeer, or other details of his personal life. From the writings of Bhāskara I (A.D. 629), it appears that Āryabhaṭa I took up, as was expected of him, the profession of a teacher. Bhāskara I mentions the names of Pāṇḍu-raṅga-svāmī, Lāṭadeva and Niśaṅku amongst those who learnt astronomy at the feet of Āryabhaṭa I. Of these pupils of Āryabhaṭa I, Lāṭa-deva is the most important and deserves special notice. He earned a name as a great scholar and teacher of astronomy. Bhāskara I has called him *Ācārya* ('Learned Teacher') and *Sarva-siddhānta-guru* ('teacher of all systems of astronomy' or 'well versed in all systems of astronomy'). From the writings of Varāhamihira (died A.D. 587) and Śrīpati (A.D. 1039), we learn that Lāṭadeva was the author of at least two works on astronomy; in one, the day was measured from midnight at Laṅkā (lat. 0, long. 75°.43 E). Varāhamihira has also

1. See *Grahacāranibandhana*, edited by K.V. Sarma, introduction, p. v. The same is stated in the *Sadratnamālā* of Śaṅkaravarman (A.D. 1800-38). See *A history of the Kerala school of Hindu astronomy*, by K.V. Sarma, p. 8.

2. *Ibid*, p. xv.

ascribed to him the authorship of two commentaries, one on the *Romaka-siddhānta* and the other on the *Pauliśa-siddhānta*. According to the Persian scholar Al-Bīrūnī (A.D. 973 to A.D. 1048), Lāṭadeva was the author of a *Sūrya-siddhānta*. Reference to 'Ācārya Lāṭadeva' has been made by Brahmagupta (A.D. 628) and his commentator Pṛthūdaka (A.D. 860) too. Pṛthūdaka has also quoted a number of verses from some work of Lāṭadeva. These verses are in *āryā* metre and their language and style are similar to those of the *Āryabhaṭīya*.

5. ĀRYABHAṬA'S WORKS

Āryabhaṭa I wrote at least two works on astronomy :

1. *Āryabhaṭīya*
2. *Āryabhaṭa-siddhānta*.

The former is well known ; the latter is known only through references to it in later works.

Varāhamihira has distinguished the two works by the reckoning of the day adopted in them. "Āryabhaṭa said," writes he,[1] "that the day begins at midnight at Laṅkā ; the same (Āryabhaṭa) again said that the day begins from sunrise at Laṅkā." Other differences between the two works of Āryabhaṭa I have been noted by Bhāskara I in this *Mahā-Bhāskarīya* (vii. 21-35).[2]

6 THE *ĀRYABHAṬĪYA*

6.1. Its contents

The *Āryabhaṭīya* deals with both mathematics and astronomy. It contains 121 stanzas in all, and is marked for brevity and conciseness of composition. At places its style is aphoristic and the case-endings are dispensed with. Like the *Yoga-darśana* of Patañjali, the subject matter of the *Āryabhaṭīya* is divided into 4 chapters, called *Pāda* (or Section).

Pāda 1 (*viz.*, *Gītikā-pāda*), consisting of 13 stanzas (of which 10 are in *gītikā* metre), sets forth the basic definitions and important astronomical parameters and tables. It gives the definitions of the larger units of time (*Kalpa*, *Manu* and *yuga*), the circular units (sign,

1. See *PSi*, xv. 20.
2. Se infra, Tables 1-5, under Sn. 8.1 below.

degree and minute) and the linear units (*yojana*, *nṛ*, *hasta* and *aṅgula*) ; and states the number of rotations of the Earth and the revolutions of the Sun, Moon and the planets etc. in a period of 43,20,000 years, the time and place from which the planets are supposed to have started motion at the beginning of the current *yuga* as well as the time elapsed since the beginning of the current *Kalpa* up to the beginning of *Kaliyuga*, the positions of the apogees (or aphelia) and the ascending nodes of the planets in the time of the author, the orbits of Sun, Moon and the planets including the periphery of the so-called sky, the diameters of the Earth, Sun, Moon and the planets, the obliquity of the ecliptic, and and the inclinations (to the ecliptic) of the orbits of the Moon and the planets, the epicycles of the Sun, Moon and the planets, and a table of sine-differences.

Pāda 2 (*viz. Gaṇita-pāda*), consisting of 33 stanzas, deals with mathematics. The topics dealt with are the geometrical figures, their properties and mensuration ; problems on the shadow of the gnomon ; series ; interest ; and simple, simultaneous, quadratic and linear indeterminate equations. The arithmetical methods for extracting the square root and the cube root and rules meant for certain specific mathematical problems including the method of constructing the sine table are also given.

Pāda 3 (*viz. Kālakriyā-pāda*), containing 25 stanzas, deals with the various units of time and the determination of the true positions of the Sun, Moon and the planets. It gives the divisions of the year (month, day, etc.) and those of the circle ; describes the various kinds of year, month and day ; defines the beginning of the time-cycle, the so-called circle of the sky, and the lords of hours and days ; explains the motion of the Sun, Moon and the planets by means of eccentric circles and the epicycles ; and gives the method for computing the true longitudes of the Sun, Moon and the planets.

Pāda 4 (*viz. Gola-pāda*), consisting of 50 stanzas, deals wtih the motion of the Sun, Moon and the planets on the celestial sphere. It describes the various circles of the celestial sphere and indicates the method of automatically rotating the sphere once in twenty-four hours ; explains the motion of the Earth, Sun, Moon and the planets ; describes the motion of the celestial sphere as seen by those on the

equator and by those on the north and south poles ; and gives rules relating to the various problems of spherical astronomy. It also deals with the calculation and graphical representation of the eclipses and the visibility of the planets.

6.2. A collection of two compositions

The *Āryabhaṭīya* is generally supposed to be a collection of two compositions : (1) *Daśagītikā-sūtra* (Aphorisms in 10 *gītikā* stanzas), which consists of *Pāda* 1, stating the astronomical parameters in 10 stanzas in *gītikā* metre, and (2) *Āryāṣṭaśata* (108 stanzas in *āryā* metre) or *Āryabhaṭa-tantra* (Āryabhaṭa's *tantra*), which consists of the second, third and fourth *Pādas*, containing in all 108 stanzas in *āryā* metre. It is noteworthy that the *Daśagītikā-sūtra* and the *Āryāṣṭaśata* both begin with an invocatory stanza and end with a concluding stanza in praise of the work and look like two different works. The commentator Bhāskara I (A.D. 629) regards the two as two different works and designates his commentaries on them by the names *Daśagītikā-sūtra-vyākhyā* and *Āryabhaṭa-tantra-bhāṣya*, respectively. He has also referred to the *Daśagītikā-sūtra* as *svatantrāntara* (author's own *tantrāntara*) in the *Āryabhaṭa-tantra-bhāṣya*.[1] Other commentators of the *Āryabhaṭīya*, too, hold the same opinion. The commentator Sūryadeva (*b.* A.D. 1191) has called the *Daśagītikā-sūtra* and the *Āryāṣṭaśata* as two compositions (*nibandhanadvaya*). The commentator Raghunātha-rāja (A.D. 1597) has also made similar statements. The commentators Yallaya (A.D. 1480) and Nīlakaṇṭha (A.D. 1500) have commented upon the second, third and fourth chapters of the *Āryabhaṭīya* only, which shows that they regarded these chapters as forming one complete work. The north Indian astronomer Brahmagupta (A.D. 628) has also referred to *Pāda* 1 of the *Āryabhatīya* as *Daśagītikā* and the rest of the *Āryabhaṭīya* as *Āryāṣṭaśata*.

It seems that the *Daśagītikā-sūtra*, which begins with an invocatory stanza and ends with a concluding stanza in praise of it, was issued as a separate tract, like the multiplication tables of arithmetic, and was meant for the freshers who were expected to learn the astronomical parameters given therein by heart before embarking upon the study of

1. See Part II, p. 188.

astronomy proper. The *Āryāṣṭaśata* was meant for those who had mastered the *Daśagītikā-sūtra* and were qualified for the study of astronomy proper.

There is no doubt, however, that the *Daśagītikā-sūtra* and the *Āryāṣṭaśata*, taken together, form the *Āryabhaṭīya* and that the *Daśagītikā-sūtra*, the *Gaṇita*, the *Kālakriyā*, and the *Gola* form the four chapters of the *Āryabhaṭīya*. This is quite clear from the following stanza of the *Daśagītikā-sūtra* where the author proposes to deal in that work three topics, *viz.*, *gaṇita*, *kālakriyā* and *gola* :

> "Having paid obeisance to Brahmā —who is one and many, the real God, the Supreme Brahman—Āryabhaṭa sets forth the three, *viz.*, mathematics (*gaṇita*), reckoning of time (*kālakriyā*) and celestial sphere (*gola*)."

Moreover, the four chapters are generally known as *Gītikā-pāda*, *Gaṇita-pāda*, *Kālakriyā-pāda*, and *Gola-pāda*, respectively. The word *pāda* means quarter or one fourth, and unless there are four chapters in a book its chapters cannot be rightly called *Pādas* or 'Quarters'.

It is noteworthy that Āryabhaṭa I himself has called *Gītikā-pāda* by the name *Daśagītikā-sūtra* and the whole work by the name *Āryabhaṭīya*. The names *Āryāṣṭaśata* and *Āryabhaṭa-tantra* were given to the second, third and fourth *Pādas* by later writers. The former occurs for the first time in the *Brāhma-sphuṭa-siddhānta* of Brahmagupta ; the latter seems to be due to Bhāskara I.

6.3. A work of the Brahma school

From the obeisance to Brahmā in the opening stanzas of the first and second *Pādas* of the *Āryabhaṭīya*, it is evident that Āryabhaṭa I was a follower of the Brahma school of Hindu astronomy. Acknowledgement of His grace at the successful completion of the *Āryabhaṭīya* in one of the closing stanzas of this work shows how deeply was he devoted to Him. This devotion to God Brahmā has led people to suppose that Āryabhaṭa I acquired his knowledge of astronomy by performing penance in propitiation of God Brahmā. In his commentary on the *Āryabhaṭīya* (i. 2), Bhāskara I writes :

> "This is what one hears said : This Ācārya worshipped God Brahmā by severe penance. So, by His grace was revealed

to him the true knowledge of the subjects pertaining to the true motion of the planets. It is said : '(Āryabhaṭa) who exactly followed into the footsteps of (Vyāsa) the son of Parāśara, the ornament among men, who, by virtue of penance, acquired the knowledge of the subjects beyond the reach of the senses and the poetic eye capable of doing good to others'."

Āryabhaṭa's devotion to Brahmā was indeed of a high order. For, in his view, the end of learning was the attainment of the Supreme Brahman and this could be easily achieved by the study of astronomy. In the closing stanza of the *Daśagītikā-sūtra*, he says :

> "Knowing this *Daśagītikā-sūtra*, the motion of the Earth and the planets, on the celestial sphere, one attains the Supreme Brahman after piercing through the orbits of the planets and the stars."

Āryabhaṭa I's predilection for the Brahma school of astronomy may have been inspired by two main considerations. Firstly, the Brahma school was the most ancient school of Hindu astronomy promulgated by God Brahmā himself. Secondly, the astronomers of Kusumapura, where Āryabhaṭa I lived and wrote his *Āryabhaṭīya*, were the followers of that school. "The learned people of Kusumapura", writes Bhāskara I, "held the *Svāyambhuva-siddhānta* in the highest esteem, even though the *Pauliśa*, the *Romaka*, the *Vāsiṣṭha* and the *Saurya Siddhāntas* were (known) there."

6.4. Its notable features

The following are the notable features of the *Āryabhaṭīya* :[3]

1. *The alphabetical system of numeral notation.* (i. 2)

The alphabetical system of numeral notation defined by Āryabhaṭa I is different from the so-called *kaṭapayādi* system but much more effective in expressing number briefly in verse.

According to this notation—

ख्युघृ	denotes the number		43,20,000
चयगियिङुशुछ्लृ	,,	,,	5,77,53,336
ङिशिबुण्लृष्खृ	,,	,,	1,58,22,37,500

For details see below, *Gītikā-pāda*, vs. 2, p. 3-5.

2. *Circumference-diameter ratio, viz.,* $\pi = 3.1416$. (ii. 10)

Āryabhaṭa I states that

$$\text{Circumference} : \text{diameter} = 62832 : 20000,$$

which is equivalent to saying that $\pi = 3{\cdot}1416$.

This value of π is correct to four decimal places and is better than the value 3·141666 given by the Greek astronomer Ptolemy.[1] It does not occur in any earlier work on mathematics and constitutes a marvellous achievement of Āryabhaṭa I.

It is noteworthy that Āryabhaṭa I has called this value only 'approximate'.

3. *The table of sine-differences.* (i. 12)

Āryabhaṭa I is probably the earliest astronomer to have given a table of sine-differences. He has also stated geometrical and theoretical methods for constructing sine-tables. For details see below *Gaṇita-pāda*, vss. 11 and 12, pp. 45-54.

4. *Formulae for* $\sin\theta$, *when* $\theta > \pi/2$. (iii. 22)

Āryabhaṭa I uses the following formulae :

$$\sin(\pi/2+\theta) = \sin\pi/2 - \text{versin}\,\theta$$
$$\sin(\pi+\theta) = \sin\pi/2 - \text{versin}\,\pi/2 - \sin\theta$$
$$\sin(3\pi/2+\theta) = \sin\pi/2 - \text{versin}\,\pi/2 - \sin\pi/2 + \text{versin}\,\theta.$$

These formulae were later used by Brahmagupta also,[2] evidently under the influence of Āryabhaṭa I.

5. *Solution of indeterminate equations of the following types :*

(*i*) $N = ax+b = cy+d = ez+f = \ldots\ldots\ldots$

(*ii*) $(ax \mp c)/b =$ *a whole number.*

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1. See Sir Thomas Heath, *A History of Greek Mathematics*, vol. 1, p. 233 ; and D. E. Smith, *History of Mathematics*, vol. 2, p. 308.

2. See *BrSpSi*, ii. 15-16.

Āryabhaṭa I is the earliest to have given the general solution of problems of the following types which reduce to the solution of the above equations :

(i) Find the number which yields 5 as the remainder when divided by 8, 4 as the remainder when divided by 9, and 1 as the remainder when divided by 7.

(ii) 16 is multiplied by a certain number, the product is diminished by 138, and the difference thus obtained being divided by 487 is found to be exactly divisible. Find the multiplier and the quotient.

For Āryabhaṭa I's solution, see below, *Gaṇita-pāda*, vss. 32-33, p. 75 ff.

6. *Theory of the Earth's rotation*

It was generally believed that that the Earth was stationary and lay at the centre of the universe and all heavenly bodies revolved round the Earth. But Āryabhaṭa I differed from the other astronomers and held the view that the Earth rotates about its axis and the stars are fixed in space. The period of one sidereal rotation of the Earth according to Āryabhaṭa I is $23^h\ 56^m\ 4^s{\cdot}1$.[1] The corresponding modern value is $23^h\ 56^m\ 4^s{\cdot}091$.[2] The accuracy of Āryabhaṭa I's value is remarkable.

7. *The astronomical parameters*

The astronomical parameters given by Āryabhaṭa I differ from those of the other astronomers and are based on his own observations. They are much better than those given by the earlier astronomers. The method used by Āryabhaṭa I for their determination has been indicated by him in the *Gola-pāda* (vs. 48).

For Āryabhaṭa I's astronomical parameters, see *Gītikā-pāda*.

8. *Time and divisions of time*

Āryabhaṭa I does not believe in the theory of creation and annihilation of the world. For him, time is a continuous process, without

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1. See *Gītikā-pāda*, vs. 3.

2. See W. M. Smart, *Text-book on Spherical Astronomy*, Cambridge, 1923, p. 492.

beginning and end (*anādi* and *ananta*). The beginnings of the *yuga* and Kalpa, according to him, have nothing to do with any terrestrial occurrence; they are purely based on astronomical phenomena depending on the positions of the planets in the sky.

In the *Smṛtis* as also in the *Sūrya-siddhānta*, we have the following pattern of time-division :

1 *Kalpa* = 14 *Manus*

1 *Manu* = 71 *yugas*

1 *Yuga* = 43,20,000 years.

In order to make the *Kalpa* equivalent to 1000 *Yugas* (in round number), every *Manu* is supposed to be preceded and followed by a period of 2/5 of a *yuga*, called twilight. A period of 3·95 *yugas* is further earmarked for the time spent in the creation of the world, so that when the world order starts all planets occupy the same place.

A Kalpa is defined as a day of Brahmā, 2 Kalpas as a nycthemeron (day and night) of Brahmā, 720 Kalpas as a year of Brahmā, and 100 years of Brahmā (or 72,000 Kalpas) as the lifespan of Brahmā. The age of Brahmā, according to the *Sūrya-siddhānta*, at the beginning of the current Kalpa, was 50 years. The current Kalpa is the first day of the 51st year of Brahmā's life, and 6 Manus with their twilights and $27\frac{9}{10}$ *yugas* had elapsed at the beginning of Kaliyuga since the beginning of the current Kalpa.

Moreover, a *yuga* is taken to be composed of 4 smaller *yugas* bearing the names Kṛta, Tretā, Dvāpara and Kali. The lengths of these smaller *yugas* are supposed to be 17,28,000; 12,96,000; 8,64,000 and 4,32,000 years, respectively.

Āryabhaṭa I rejects this highly artificial scheme of time-division, and replaces it by the following :

1 day of Brahmā or *Kalpa* = 14 *Manus* or 1008 *yugas*

1 *Manu* = 72 *yugas*

1 *yuga* = 43,20,000 years.

Āryabhaṭa I has dispensed with the periods of twilight and the time spent in creation, and has simplified the scheme enormously. Since $1008 \equiv 0 \pmod 7$, every *Kalpa* under this scheme begins on the same day, which is an additional advantage. Under this scheme, 6 *Manus* and $27\frac{3}{4}$ *yugas* had elapsed at the beginning of the current Kaliyuga since the beginning of the current *Kalpa.*

Āryabhaṭa, too, divided a *yuga* into 4 smaller *yugas*, but he takes them to be of equal duration and calls them quarter-*yugas*, the duration of each being 10,80,000 years. This is indeed a more scientific division, because in every quarter *yuga* the planets make an integral number of revolutions round the Earth.

Although the time-divisions given in the *Sūrya-siddhānta* and by Āryabhaṭa I differ so much, they have been so adjusted that the beginning of the current Kaliyuga according to both of them falls on the same day, *viz.*, Friday, February, 18, 3102 B.C.

Āryabhaṭa I has also divided his *yugas* into 2 divisions, *Utsarpiṇī* and *Apasarpiṇī*; and further *Utsarpiṇī* into *Duṣṣamā* and *Suṣamā*, and *Apasarpiṇī* into *Suṣamā* and *Duṣṣamā*, respectively.[1] This is evidently under the influence of the Jaina scholars of Pāṭaliputra where Āryabhaṭa I lived. Pāṭaliputra, was the original home of the Jainas and a bulwark of Jaina saints and scholars in ancient times.

9. *Theory of the planetary motion*

The computation of the planetary positions in the *Āryabhaṭīya* is based on the following hypotheses :

Hypothesis 1. In the beginning of the current *yuga*, which occurred on Wednesday, 32,40,000 years before the commencement of the current quarter-*yuga*, all the planets together with the Moon's apogee and the Moon's ascending node were in conjunction at the first point of the asterism Aśvinī (ζ Piscium). (*Gītikā-pāda*, vs. 4 (d)).

Hypothesis 2. The mean planets revolve in geocentric circular orbits.

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1. For details, see below, *Kālakriyā*, vs. 9, pp. 93-94.

The mean motions of the planets are given in terms of revolutions performed by the planets round the Earth in a period of 43,20,000 years. These revolutions, as already stated, are based on Āryabhaṭa I's own observations, and constitute the main distinguishing feature of Āryabhaṭa I's astronomy. For details see *Gītika-pada*, vss. 3-4.

Hypothesis 3. The true planets move in eccentric circles or in epicycles.

For details see *Kālakriyā-pāda*, vss. 17-21.

The eccentric and the epicyclic theories of Āryabhaṭa I have been explained in greater detail by Bhāskara I in his *Mahā-Bhāskarīya* (ch. iv). Two things may, however, be mentioned here :

(i) The *manda* epicycles are not the actual epicycles but the mean epicycles corresponding to the mean distances of the planets.

(ii) The radius of the *śīghra* concentric (and therefore of the *śīghra* eccentric), according to Āryabhaṭa I, is equal to the planet's distance called *mandakarṇa*.

The Greek astronomer Ptolemy, too, explained the motion of the planets with the help of epicycles and eccentric circles, but the method used by Āryabhaṭa I for explaining the planetary motion is quite different and much simpler than that used by Ptolemy. Bina Chatterji who has made a comparative study of the Greek and the Hindu epicycles and eccentric theories, concludes that "Āryabhaṭa's epicyclic and eccentric methods are unaffected by Ptolemaic ideas".[1]

It may be pointed out that whereas the epicycles of Ptolemy are of fixed dimensions, those of Āryabhaṭa I vary in size from place to place. The variable (or pulsating) epicycles probably yielded better results. The later Hindu astronomers have followed Āryabhaṭa I in taking variable epicycles.

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1. See Bina Chatterjee, ***The Khaṇḍa-khādyaka of Brahmagupta***, **Vol. I, Appendix VII, p. 293. World Press, Calcutta, 1970.**

Hypothesis 4

All planets have equal linear motion in their respective orbits. (*Kālakriyā-pāda*, vs. 12).

10. *Innovations in planetary computation*

From the old *Sūrya-siddhānta*, summarized by Varāhamihira, and from the *Sumati-mahātantra* of Sumati, we learn that the earlier astronomers performed four corrections in the case of the superior planets (Mars, Jupiter and Saturn) and as many as five corrections in the case of the inferior planets (Mercury and Venus) in order to obtain their true positions.[1] The fifth correction applied to the inferior planets was again purely impirical in character and was artificially devised to get correct results. Āryabhaṭa changed the old pattern of correction and, in the case of the inferior planets, reduced the number of corrections from five to three. In the the case of the superior planets, too, the corrections were the same, but a pre-correction (equal to half the equation of centre) was also prescribed. This innovation was an improvement and yielded more accurate results.

In the case of finding planetary distances, the *Sūrya-siddhānta*[2] prescribed the formula

$$\frac{mandakarṇa + śīghrakarṇa}{2}$$

but Āryabhaṭa[3] changed it to

$$\frac{mandakarṇa \times śīghrakarṇa}{R}$$

11. *Celestial latitudes of the planets*

In the time of Āryabhaṭa, astronomers were helpless in finding the celestial latitudes of the planets. The methods given in the old *Sūryasiddhānta*,[4] summarised by Varāhamihira, and even in the *Siddhānta*

1. See *PSi*, xvi. 17-22. Also see K. S. Shukla, The *Pañcasiddhāntikā* of Varāhamihira (I), *IJHS*, vol. ix, no. 1, 1974, pp. 62-76.

2. vii. 14.

3. See *Ā*, iii. 25.

4. See *PSi*, xvi. 24-25.

of Āryabhaṭa himself,[1] (which was written earlier than the *Āryabhaṭīya*), are not correct. It is Āryabhaṭa who in his *Āryabhaṭīya*[2] for the first time gave the correct method for finding the celestial latitude of planets, both superior and inferior.

12. *Use of the radian measure in minutes*

Āryabhaṭa is probably the earliest astronomer to use the radian measure of 3438′ for the radius of his circles. His table of Rsine-differences is also given in the same measure. Measurement of the radius in minutes facilitates computation and most of the astronomers in India have followed Āryabhaṭa in this respect. Brahmagupta (A.D. 628) who did not use this measure in his Rsine-table was criticised by Vaṭeśvara (A.D. 904).

7·5. Its importance and popularity

Brevity and conciseness of expression, superiority of astronomical constants, and innovations in astronomical methods rendered the *Āryabhaṭīya* an excellent text-book on astronomy. It gave birth to a new school of astronomy, the Āryabhaṭa School, whose exponents called themselves 'disciples of Āryabhaṭa'. These disciples of Āryabhaṭa deified Āryabhaṭa I as 'Bhagavān' and 'Prabhu' and held the teachings of the *Āryabhaṭīya* in the highest esteem, claiming greater accuracy for them. Bhāskara I, writing in the first half of the seventh century A.D., declares : "None except Āryabhaṭa has been able to know the motion of the heavenly bodies : others merely move in the ocean of utter darkness of ignorance." Bhāskara I was the most competent exponent of the Āryabhaṭa school. He wrote a commentary on the *Āryabhaṭīya* and two other works on astronomy in illucidation of the teachings of Āryabhaṭa I. He earned a great name as a teacher of astronomy and was well known as 'guru'. The works of Bhāskara I throw a flood of light on the astronomical theories and methods of Āryabhaṭa I and on the earlier followers of Āryabhaṭa I. His commentary on the *Āryabhaṭīya*, which was utilized by most of the subsequent commentators, was recognized as a work of great scholarship and its author came to be designated as 'all-knowing commentator'.

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1. See *MBh*, vii. 28(c-d)-33.
2. iv. 3.

The works of Bhāskara I provided a great stimulus to the study of the *Āryabhaṭīya* which became a popular work and continued to be studied at various centres of learning in South India, especially in Kerala, till recent times. The extent of the popularity enjoyed by the *Āryabhaṭīya* can be easily estimated by the following facts : (1) There is hardly any work dealing with Hindu astronomy which does not refer to Āryabhaṭa or quote from the *Āryabhaṭīya*. (2) There exist a number of commentaries on the *Āryabhaṭīya* written in Sanskrit and other regional languages by authors hailing from far-flung places in South India. (3) There exist a number of independent astronomical works which are based on the *Āryabhaṭīya*. (4) Calendrical texts and tables used in South India after the first half of the seventh century A.D. until the introduction of new works based on the western astronomical tables and the *Nautical Almanac* were based on the *Āryabhaṭīya* or works based on it.

In northern India, too, the *Āryabhaṭīya* continued to be studied at least up to the end of the tenth century A.D. Brahmagupta, who lived in the seventh century at Bhinmal in Rajasthan, made an intensive study of this work. He utilized this work in writing his *Brāhma-sphuṭa-siddhānta*, and a number of passages in that work are strikingly similar to those found to occur in the *Āryabhaṭīya*. Pṛthūdaka, who lived at Kannauj in Uttar Pradesh, quotes, in his commentary on the *Brāhma-sphuṭa-siddhānta* written in A.D. 860, a number of passages from the *Āryabhaṭīya*. It is remarkable that for finding the volume of a sphere, Pṛthūdaka prescribes exactly the same rule as found in the *Āryabhaṭīya*. He has evidently taken it from the *Āryabhaṭīya*, since this rule is typically Āryabhaṭa's and is not found to occur in any other work on Indian astronomy. Several passages from the *Āryabhaṭīya* occur in the writings of the Kashmirian scholar Bhaṭṭotpala who wrote about A.D. 968.

7·6. Commentaries on the *Āryabhaṭīya*

(a) *Commentaries in Sanskrit*

1. Bhāskara I's commentary

Bhāskara I's commentary on the *Āryabhaṭīya* has been critically edited in Part II of this *Series*. It is the earliest commentary on the *Āryabhaṭīya* that has come down to us. Written at Valabhī in Sau-

rāṣṭra (modern Kathiawar) in the year A.D. 629, it sets forth a comprehensive exposition of the contents of the *Āryabhaṭīya*. "These who want to know everything written by Āryabhaṭa", writes Śaṅkaranārāyaṇa (A.D. 869), "should read the commentary on the *Āryabhaṭīya* and the *Mahā-Bhāskarīya* (written by Bhāskara I)."

2. Prabhākara's commentary

From two passages in Bhāskara I's commentary on the *Āryabhaṭīya* it appears that Prabhākara was an earlier commentator of the *Āryabhaṭīya*. In both the places, Bhāskara I finds fault with the interpretations given by Prabhākara. Bhāskara I calls him 'Ācārya Prabhākara', but says: "He is a teacher, bethinking thus I am not censuring him." This Prabhākara may have been the same person as has been called 'a disciple (follower) of Āryabhaṭa' by Bhāskara II (A.D. 1150) in his commentary on the *Śiṣya-dhī-vṛddhida* of Lalla. Ācārya Prabhākara has also been mentioned by Śaṅkaranārāyaṇa (A.D. 869), Udaya-divākara (A.D. 1073), Sūryadeva (*b.* A.D. 1191) and Nīlakaṇṭha (A.D. 1500). Prabhākara's commentary has not survived the ravages of time, nor has it been mentioned by any later writer.

It is noteworthy that Sūryadeva (*b.* A.D. 1191), in his commentary on the *Laghu-mānasa* (iv. 2), refers to Prabhākara as 'Prabhākara-guru' and mentions his work *Prabhākara-gaṇita*. It is not known whether this Prabhākara was the same person as one criticised by Bhāskara I.

3. Someśvara's commentary

A manuscript of Someśvara's commentary on the *Āryabhaṭīya* exists in the Bombay University Library, Bombay.[1] The beginning and end of it are as follows :

Beginning : श्री गणेशाय नम:

अथ आर्यभटसिद्धान्त: समाष्य: प्रारभ्यते ।

नमो नवग्रहेभ्य: ।

सर्वहितं सर्वज्ञं प्रणम्य जगदर्चितं शिवं भक्त्या ।
वृत्तिमहं संक्षेपात् वच्मि स्पष्टां च शिष्यहिताय ॥

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1. Catalogue No. 335, Bookshelf No. 272, Accession No. 2495.

तवत्राचार्यार्यभटवदनकमलोद्गतगीतिकासूत्रस्यादौ निःशेषविघ्नविनाशाय भगवतः कमलयोनेः नमस्कारः—

प्रणिपत्य etc.

End : स्पष्टार्थप्रतिपादकं मृदुधियां सूक्तं प्रबोधप्रदं
तर्कव्याकरणाविशुद्धमतिना सोमेश्वरेणाधुना ।
आचार्यार्यभटोक्तसूत्रविवृतिर्या भास्करोत्पादिता
तस्याः सारतरं विकृष्य रचितं भाष्यं प्रकृष्टं लघु ॥

Colophon : इति सोमेश्वरविरचितमार्यभटीयभाष्यं समाप्तम् ।

The contents of this commentary show that, as acknowledged by the author himself in the closing stanza, it is a summary of Bhāskara I's commentary. Even the introductory lines given before the verses commented upon are sometimes almost exactly the same as found in Bhāskara I's commentary. In the commentary on the *Gaṇitapāda*, however, Someśvara has set some new examples besides those taken from the commentary of Bhāskara I.

Someśvara's commentary does not throw any light on the life and works of its author.

A commentary by Someśvara on the *Khaṇḍa-khādyaka* of Brahmagupta is mentioned by Āmarāja in the opening stanza of his commentary on the same work. From the order in which Āmarāja mentions the names of the earlier commentators of the *Khaṇḍa-khādyaka*, it appears that Someśvara lived posterior to Bhaṭṭotpala (A. D. 968). Since Āmarāja lived about 1200 A.D., Someśvara must have lived sometime between A.D. 968 and A.D. 1200.

4. Sūryadeva Yajvā's commentary

Sūryadeva calls himself Sūryadeva Yajvā, Sūryadeva Somasut and sometimes Sūryadeva Dīkṣita.

Sūryadeva's commentary on the *Āryabhaṭīya* has been critically edited in Part III of this *Series*. It is usually known by the following names : *Āryabhaṭa-prakāśa*, *Bhaṭa-prakāśa*, *Prakāśa*, *Āryabhaṭa-prakāśikā*, *Bhaṭa-prakāśikā* and *Prakāśikā*.

Sūryadeva's commentary sets forth an excellent exposition of the *Āryabhaṭīya*. It has been illucidated by further notes and examples by

Yallaya (A.D. 1480) and has been used as a source book by Parameśvara (A.D. 1431) in writing his own commentary on the *Āryabhaṭīya*.

Sūryadeva is the author of at least five commentaries, which he wrote in the following order :

(1) An exposition of Govinda-svāmī's *bhāṣya* on the *Mahā-Bhāskarīya* of Bhāskara I (A.D. 629).

(2) Commentary on the *Āryabhaṭīya*

(3) Commentary on the *Mahā-yātrā* of Varāhamihira.

(4) Commentary on the *Laghu-mānasa* of Mañjula (A.D. 932).

(5) Commentary on the *Jātaka-paddhati* of Śrīpati (A.D. 1039).

From his commentary on the *Laghu-mānasa* of Mañjula (A.D. 932), we learn that :

(1) Sūryadeva was born on Monday, 3rd *tithi* of the dark half of Māgha, Śaka 1113 (=A.D. 1191). The *ahargaṇa* for that day, according to the *Āryabhaṭa-siddhānta*, was 15,68,004.[1]

(2) He was a Brāhmaṇa of Nidhruva *gotra*.[2]

(3) He belonged to the Cola country (which roughly comprised of Tanjore and Trichinopoly districts of Tamilnadu) and was the resident of the town called by the names Gaṅgāpura, Gaṅgāpurī and Śrīraṅga-gaṅgāpurī[3] which may be easily identified with Gaṅgai-koṇḍa-Coḷapuram (lat. 11° 13′ N.,

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1. Sūryadeva writes : विश्वेशमिते 1113 शाके माघकृष्ण-तृतीयायां सोमवारे आचार्यार्यभटसिद्धान्तसिद्धोऽस्मज्जन्मदिनेऽहर्गणः 15,68,004.

2. See colophons at the ends of chs. i and ii.

3. *Cf.*, चोलदेशे गङ्गापुरे (पलभा) अङ्गुलद्वयमङ्गुलषष्टिभागाश्चतुर्विंशतिः (*LMā*, ii. 1 com.); चोलदेशे गङ्गापुरे सिद्धाश्चरगुणाः क्रमेण 48, 38, 16 (*LMā*, ii. 1 com.); गङ्गापुरे चरगुणानामर्धानि क्रमेण 24, 19, 8 । एतैः क्रमेण हीना वसुभादयः मेषस्य 254, वृषस्य 280, मिथुनस्य 315········एवमेतानि गङ्गापुरे मेषादिद्वादशराश्युदय-विनाडीमानानि भवन्ति (*LMā*, ii. 2 com.) ; तदिदमुदाहरणेन गङ्गापुर्यां प्रदर्श्यते··· गङ्गापुरेऽगस्त्योदयलग्नम् ·· गङ्गापुरेऽस्तं याति (*LMā*, iv. 4 com.) ; चोळदेशे श्रीरङ्गगङ्गापुर्याः खरनगरस्य चान्तरे योजनात्मकोऽध्वा 11, ··· ··· श्रीरङ्गगङ्गापुर्यां चन्द्रे क्षयं कृत्वा (*LMā*, i (c). 3, com.). It may be mentioned that, out of the two manuscripts consulted, one has but the other does not have the word श्रीरङ्ग prefixed to गङ्गापुर्याः and गङ्गापुर्यां in this last reference.

long. 79° 30′ E.),[1] for, according to Sūryadeva, the equinoctial midday shadow at that place was $2\frac{2}{5}$ *aṅgulas* which corresponds to the latitude of 11°·3 N. This is also substantiated by the ascensional differences and times of risings of the signs stated by Sūryadeva for the said place. The distance of that place from the Hindu prime meridian is said to have been 11 *yojanas* eastwards.

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1. Gaṅgai-koṇḍa-Coḷapuram ('the city of the Cola king, who conquered up to the Ganges'), also called Gaṅgā-koṇḍa-puram, is a town and temple in Trichinopoly district of Tamilnadu. It is located between Colaroon (a branch of the river Cauveri at its delta region) and the river Veḷḷār flowing on its two sides and is situated about six miles from Jayamkoṇḍa-Śolapuram. It is connected with Uḍaiyār-pāḷaiyam by the Chidambaram road, and is one mile distant from the great Trunk Road running from Tanjore to South Arcot.

Gaṅgai-koṇḍa-Coḷapuram was founded by king Rājendra Cola I (A.D. 1012-44) who was called 'Gaṅgai-koṇḍa-Coḷa' (lit. 'The Coḷa who conquered up to the Ganges') and who shifted his capital from Tanjore to this city, and was known after his name 'the city of the Cola king, who conquered up to the Ganges'. This city remained the capital of the Coḷa kings for many years to come. It has now lost its past glory and is no more than a village. Close to it stand the ruins of one of the most remarkable but least known temples in Southern India. The temple consists of one large enclosure, measuring 584 feet by 372 feet. The *vimāna* in the centre of the courtyard is a very conspicuous building and strikes the eye from a great distance. The pyramid surrounding it reaches a height of 174 feet. The ruins of six *gopuras*, or gate-pyramids, surmount different parts of the building. That over the eastern entrance to the main enclosure, was evidently once a very fine structure, being built entirely of stone except at the very top. All the lower part of the central building is covered with inscriptions.

Gaṅgai-koṇḍa-Coḷapuram was called Gaṅgāpurī in Sanskrit (*Cf. Epigraphia Indica*, xv, p. 49). The word *Śrī-raṅga* (literally meaning 'the stage of the goddess of prosperity') prefixed to the name Gaṅgāpurī in one of the manuscripts consulted seems to point to the richness and magnificence of this city.

(4) He wrote his commentary on the *Laghu-mānasa* in A.D. 1248 (*i.e.*, at the age of 57 years). This is inferred from the fact that Sūryadeva has stated the *dhruvakas* (planetary positions) for Thursday noon, Caitrādi, Śaka year 1170.

5. Parameśvara's commentary

Parameśvara's commentary on the *Āryabhaṭīya* was edited by H. Kern and printed at Leiden (Holland) in A.D. 1874. It was reprinted in A.D. 1906 by Udaya Narain Singh along with his Hindi translation of the *Āryabhaṭīya.*

Parameśvara's commentary sets forth a brief but excellent exposition of the *Āryabhaṭīya.* In writing this commentary the author has utilized Sūryadeva's commentary, and has quoted from the *Sūrya-siddhānta,* the *Brāhma-sphuṭa-siddhānta* of Brahmagupta, the *Bṛhat-saṁhitā* of Varāhamihira, the *Śiṣya-dhī-vṛddhida* of Lalla, the *Triśatikā* of Śrīdhara, and the *Līlāvatī* of Bhāskara II. He has also referred to his *Mahābhāskarīya-bhāṣya-vyākhyā Siddhāntadīpikā,* which was written sometime after A.D. 1431. His commentary on the *Āryabhaṭīya* was evidently a later work.

Parameśvara hails from Kerala. He lived in the village Aśvattha (identified with modern Ālattūr) situated on the north bank of the river Nīḷā (Mal. Bhāratappuzha) near the Arabian sea shore. His first composition was his commentary on the *Laghu-Bhāskarīya* which he wrote in A.D. 1408 when he was still a student. If we presume that he was 28 years of age at that time, his date of birth may be fixed at 1380 A.D. His *Dṛggaṇita* was written in A.D. 1431 and his *Goladīpikā* in A.D. 1443.

Parameśvara wrote a number of books on astronomy, astrology and allied subjects. See Part II, *Scholiasts of Bhāskara I.*

6. Yallaya's notes on Sūryadeva's commentary

Yallaya has written notes on Sūryadeva's commentary dealing with the second, third and fourth *Pādas* of the *Āryabhaṭīya.* Yallaya's commentary on each verse of the *Āryabhaṭīya* consists of Sūryadeva's commentary followed by Yallaya's notes where necessary. In his notes Yallaya has sometimes illustrated the rules by giving suitable examples with solutions.

A manuscript of this commentary exists in the Lucknow University Library. The colophon at the end of it runs as follows :

इति श्रीमच्चन्द्रशेखरवरलब्धवाग्विभवेन श्रीबालादित्यसुतसूर्याचार्यशिष्येण श्रीधरार्यपुत्रेण यल्लयाख्येन विपश्चिता आर्यभटोक्तगोलपादस्य किञ्चित्तात्पर्यव्याख्यानं कृतम् ।

The scope of the commentary in the words of its author is as follows :

> "As the commentary written by Sūryadeva Yajvā, who had thorough knowledge of the science of words (*i.e.*, grammar), is brief, so for the benefit of those astronomers who want to know the (detailed) meaning of the *Gaṇita, Kālakriyā* and *Gola Pādas* (of the *Āryabhaṭīya*) composed by Āryabhaṭa, I, learned Yallaya, son of Śrīdharārya, pupil of Sūryācārya son of Bālāditya, well versed in many works on *Pāṭīgaṇita* and proficient in the three branches of astronomy, and who has command over language by virtue of the boon acquired from God Śiva, will first write those entire explanations of the *ārya-sūtras* ('aphorisms in *āryā* metre') which have been given by Sūryadeva Yajvā and then, wherever the explanations are brief, will supplement them by further explanations and alternative illustrative examples."[1]

From the above passage we learn that Yallaya was a son of Śrīdharārya and a pupil of Sūryācārya.[2] This Śrīdhara was different from the author of the *Pāṭīgaṇita* and the *Triśatikā*.* And this Sūryācārya was the author of : (1) *Gaṇakānanda*, (2) *Daivajñābharaṇa* and (3) *Daivajña-bhūṣaṇa* and was different from Sūryadeva (*b.* A.D. 1191). Sūryācārya's father Bālāditya was also a famous astronomer. He was

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1. प्रचुरतरशब्दशास्त्रविदा सूर्यदेवेन यज्वना कृतस्य व्याख्यानस्य संक्षेपत्वात् आर्यभटोक्तगणितकालक्रियागोलपादार्थान् ज्ञातुमिच्छतां दैवज्ञानामुपकाराय यस्य यस्य सूत्रस्य सूर्यदेवेन यज्वना यत् व्याख्यानं कृतं (तस्य) तस्य सूत्रस्य तत् कृत्स्नं व्याख्यानं लिखित्वा यत्र यत्र व्याख्यानसंक्षेपं (तत्र) तत्र बहुपाटीगणितग्रन्थवित्त्रिस्कन्धार्थविदा श्रीमच्चन्द्रशेखरवरलब्धवाग्विभवेन श्रीधरार्यस्य पुत्रेण श्रीबालादित्यसुतसूर्याचार्यशिष्येण श्रीयल्लयाख्येन विपश्चिता मया व्याख्यानमपि किञ्चिदधिकमुदाहरणान्तराणि च क्रियन्ते ।

2. In his commentary on the *Sūrya-siddhānta* (written in A.D. 1478) Yallaya has called his teacher by the name Sūrya-Sūri and has quoted a large number of passages from the astronomical work *Daivajñābharaṇa* written by him.

the author of a work called *Bāla-Bhāskarīya* which has been often quoted by the commentators hailing from the Andhra State.

As regards the place where the present commentary was written, Yallaya himself writes :

> "This exposition was carefully composed by me in the town of Skandasomeśvara which is situated towards the south-east of Śrīśaila."[1]

The equinoctial midday shadow and the equinoctial midday hypotenuse for Skandasomeśvara are given to be $3\frac{1}{3}$ *aṅgulas* and $12\frac{7}{18}$ *aṅgulas*, respectively. From these figures the latitude of the place comes out to be 15° 30′N. approximately. The distance of Skandasomeśvara from the Hindu prime meridian is stated to be 36 *yojanas* (according to the reckoning of the *Sūrya-siddhānta*) or $23\frac{7}{18}$ *yojanas* (according to the reckoning of the *Āryabhaṭīya*),[2] which corresponds to 4° 5′. As the commentator applies the corresponding longitude correction to the planets negatively, it follows that Skandasomeśvara was situated 4° 5′ to the east of the Hindu prime meridian.[3]

Skandasomeśvara was not only the place where this commentary was written, but it was also the place to which Yallaya actually belonged. For in his commentary on the *Sūrya-siddhānta* (i. 57-58), Yallaya says :

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1. श्रीशैलस्याग्नेयस्थितस्कन्दसोमेश्वरपट्टणे वृत्तिः मया सम्यक् निर्णीतम् । (Comm. on *Ā*, iii. 6).

Śrīśaila is a temple in the Nandikotkūr tāluk of Kurnool District, Andhra State, situated in latitude 16° 5′ N. and longitude 78° 53′ E. It lies in the midst of malarious jungles and rugged hills on the northernmost plateau of the Nallamalais, overlooking a deep gorge through which flows the Krishna river. The temple is 600 feet long by 510 feet broad. The walls are elaborately sculptured with scenes from the *Rāmāyaṇa* and the *Mahābhārata*. In the centre stands the shrine of Mallikārjuna, the name by which God Śiva is worshipped there.

2. Comm. on *Ā*, iii. 6.

3. Our conclusions agree with the computations made in the ommentary.

"My native country, however, is the town called Skandasomeś-vara which lies towards the south-east of Śrīśaila."[1]

The date of writing the commentary is 1480 A.D., which corresponds to the Kali year 4581 mentioned in the commentary.[2] Other dates mentioned in the commentary are A.D. 1456, A D. 1465, A.D. 1466 and A.D. 1469.

In the present commentary there are a number of rules and examples which have been cited from the works of earlier writers. The sources are generally not mentioned. Some rules, however, can be easily traced to the *Triśatikā* of Śrīdhara. Several examples are borrowed from the commentary of Bhāskara I. Some rules and examples are Yallaya's own composition. At one place the commentator (Yallaya) refers to the people of Āndhra and Karṇāṭaka, saying that they call the number 10^{10} (*arbuda*) by the denomination *śatakoṭī*.

The following tables given by Yallaya in the commentary will be useful to historians :

1. Table of linear measures

8 *paramāṇus*	=	1 *trasareṇu*
8 *trasareṇus*	=	1 *rathareṇu*
8 *rathareṇus*	=	1 *kośa*
8 *kośas*	=	1 *tilabīja*
8 *tilabījas*	=	1 *sarṣapa*
8 *sarṣapas*	=	1 *yava*
8 *yavas*	=	1 *aṅgula*
12 *aṅgulas*	=	1 *vitasti*
2 *vitastis*	=	1 *hasta*
4 *hastas*	=	1 *daṇḍa*
2000 *daṇḍas*	=	1 *krośa*
4 *krośas*	=	1 *yojanà*

1. मदीयदेशस्तु श्रीशैलस्याग्नेयदिग्भागे स्थित: स्कन्दसोमेश्वराख्यपट्टणः ।

2. Comm. on *Ā*, iii. 6.

2. Table of grain measures

4 *kuḍubas*	=	1 *prastha*
4 *prasthas*	=	1 *āḍhā*
4 *āḍhās*	=	1 *droṇa*
5 *droṇas*	=	1 *khārī*

3. Table of gold or silver measures

4 *vrīhis*	=	1 *guñja*
2 *guñjas*	=	1 *māṣaka*
2 *māṣakas*	=	1 *gumarta*
10 *gumartas*	=	1 *suvarṇa*
1½ *suvarṇas*	=	1 *karṣa*
4 *karṣas*	=	1 *pala*

4. Names of 29 notational places

(1) *eka*, (2) *daśa*, (3) *śata*, (4) *sahasra*, (5) *ayuta*, (6) *lakṣa*, (7) *prayuta*, (8) *koṭi*, (9) *daśakoṭi*, (10) *śatakoṭi*, (11) *arbuda*, (12) *nyarbuda*, (13) *kharva*, (14) *mahā-kharva*, (15) *padma*, (16) *mahā-padma*, (17) *śaṅkha*, (18) *mahā-śaṅkha*, (19) *kṣoṇī*, (20) *mahā-kṣoṇī*, (21) *kṣiti*, (22) *mahā-kṣiti*, (23) *kṣobha*, (24) *mahā-kṣobha*, (25) *parārdha*, (26) *sāgara*, (27) *ananta*, (28) *cintya*, and (29) *bhūri*.[1]

7. Nīlakaṇṭha Somayājī's commentary

This commentary bears the name *Mahā-bhāṣya* and has been published in the *Trivandrum Sanskrit Series*, Nos. 101, 112, 185. Nīlakaṇṭha, like Yallaya, has commented only upon the *Gaṇita*, *Kāla-kriyā* and *Gola Pādas* of the *Āryabhaṭīya*.

From the colophon occurring at the end of the commentary on the *Gaṇita-pāda*, we have the following information regarding the commentator (Nīlakaṇṭha) :

1. His father was called Jātaveda, the same being the name of his maternal uncle also.

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1. In place of *parārdha* some people, says Yallaya, use the denomination *saṅkṛti*. In the list given by Mahāvīra (A.D. 850), *śaṅkha* and *mahā-śaṅkha* have been replaced by *kṣoṇī* and *mahā-kṣoṇī*, respectively, and *vice versa*. See B. Datta and A.N. Singh, *History of Hindu Mathematics*, Part I, p. 13.

2. His younger brother was named Śaṅkara.
3. He was a Brāhmaṇa, follower of the Āśvalāyana-sūtra, and belonged to the Gārgya-gotra.
4. His teacher in astronomy was Dāmodara, son of the commentator Parameśvara (A.D. 1431) ; and his teacher in Vedānta was Ravi, who was probably the same person as the author of the *Ācāradīpikā*—a commentary in verse on the *Muhūrtāṣṭaka.*
5. He was a native of the village Kuṇḍa, which has been identified with Tṛkkaṇṭiyūr in South Malabar, Kerala.

In the commentary on verses 12-15 of the *Kālakrīyā-pāda,* the commentator writes :

> "When 16,68,478 days had elapsed since the beginning of Kaliyuga, we observed a total eclipse of the Sun ; and when 16,81,272 days had elapsed, there occurred an annular eclipse in Ananta-kṣetra."

The first epoch corresponds to A.D. 1467 and the latter to A.D. 1502. Thus it is evident that this commentary was written after A.D. 1502. From the commentator's *Siddhānta-darpaṇa-vyākhyā,* we learn that he was born in December A.D. 1444. Hence at the time of writing the present commentary he was above 60 years of age.

Nīlakaṇṭha's commentary on the *Āryabhaṭīya* is a valuable work as it incorporates the advances made in astronomy up to his time and contains a good deal of matter of historical interest. There are quotations from the works of Varāhamihira, Prabhākara, Jaiṣṇava or Jiṣṇunandana (*i.e.*, Brahmagupta, son of Jiṣṇu), Bhāskara I, Muñjālaka (same as Muñjāla or Mañjula), Śrīpati, Bhāskara II, Mādhava, resident of the village Saṅgama ;[1] from the *Sūrya-siddhānta,* the *Siddhānta-dīpikā* of Parameśvara, Sūryadeva's commentary on the *Āryabhaṭīya,* Govinda-svāmī's commentary on the *Mahā-Bhāskarīya,* and from his own works, the *Golasāra* and the *Tantra-saṅgraha.* Reference is also made to Vṛddha-Garga, Garga, Paramarṣi, Maṇittha, Vyāsa, Vārttikakāra,

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1. Saṅgama-grāma is identified with Iriñjālakkuḍa or Iriṅṅālakkuḍa, near Cochin.

Piṅgala, Bhaṭṭapāda, Haradatta (same as Haridatta), Dāmodara, Kauṣītakī Netranārāyaṇa, and to the *Vākyapadīya*, the *Vaijayantī*, the *Vyāpti-nirṇaya* of Pārthasārathi Miśra, the *Laghu-Bhāskarīya* and Udayadivākara's commentary on the *Laghu-Bhāskarīya*. Passages from some of these works are also cited.

At one place in the commentary,[1] Magadha and Baudhāyana are reported to have stated in their works the amount of precession of the equinoxes for their times. The following hemistich is ascribed to the *Garga-saṁhitā* :[2]

पृथग्दोःकोटिवर्गाभ्यां कर्णवर्गोऽनुषज्यते ।

which means that if b, k, and h be the base, the upright and the hypotenuse of a right-angled triangle, then $b^2+k^2=h^2$.

8. Raghunātha-rāja's commentary

A manuscript of this commentary is available in the Lucknow University Library, Lucknow. The beginning and end of it are as follows :

श्रीरामजयम् । शुभमस्तु ।

वन्दे श्रीवदनारविन्दतरणिं श्रीगारुडाद्रीश्वरं
प्रह्लादार्चितपादपद्मयुगलं भक्तामृताब्धिं सदा ।
भाषाधीश्वरपार्वतीप्रियशचीनाथादिसंसेवितं
ज्ञानानन्दनिधिं निरस्तकलुषं वात्सल्यवारांनिधिम् ॥
यस्तन्वन् सुरवैरिसूनुवचनं स्तम्भादभूत् सूनृतं
यः श्रीवक्त्रसरोजदर्शनमुषा शान्ताकृतिस्तत्क्षणात् ।
लोकान् सामरदानवान् समतनोत् भीतान् पुराग्निर्भयान्
स श्रीगारुडशैलशेखरमणिर्देवो नृसिंहोऽवतात् ।

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1. Comm. on iii. 10.

2. Comm. on ii. 4. Garga, author of the *Garga-saṁhita* (who was different from Vṛddha-Garga), is said to have been born in the beginning of Kaliyuga. In support of this is adduced the evidence of Parāśara as also Garga's own assertion in the *Garga-saṁhitā*. *Vide* comm. on iii. 10.

कर्णाटवंशकलशाम्बुधिकौस्तुभाभो
विख्यातकीर्तिरभवद् भुवि वेङ्कटाख्यः ।
तस्यानघः शुभगुणोऽजनि नागराजः
तस्यात्मजो गुणनिधिर्भुवि कोण्डभूपः ।
तस्य श्रीपतिदेशिकेन्द्रकरुणापूर्णोऽस्ति पुत्रो महान्
नाम्ना श्रीरघुनाथराज इति यः ख्यातो विशुद्धाशयः ।
लक्ष्मीगर्भसुधाब्धिकौस्तुभमणिः ख्याताश्च यद्भ्रातरः
सन्तः कल्पतरोरिव प्रतिदिनं कामान् लभन्ते यतः ॥
करोम्यार्यभटव्याख्यां कर्णाटकुलसम्भवः ।
विशदां विदुषां प्रीत्यै रघुनाथावनीश्वरः ॥

"स्वाध्यायोऽध्येतव्यः", "स्वाध्यायमधीयीत" ...

End : एवं गोलपादोऽप्युद्देशतः व्याख्यातः । अत्र गणितपादे त्रयस्त्रिंशत् सूत्राणि । कालक्रियापादे पञ्चविंशतिः । गोलपादे पञ्चाशत् । एवमष्टोत्तरशतम् अस्मिन् प्रबन्धे । पूर्वस्मिन् प्रबन्धे त्रयोदश । एवं सूत्राणामेकविंशत्युत्तरशतमतीन्द्रियार्थप्रकाशकमाचार्येण प्रणीतम् । एतानि च दिङ्मात्रेण मया व्याख्यातानि । एवमेकैकस्य सूत्रस्य ग्रन्थसहस्रेणापि निरवशेषार्थप्रतिपादनमशक्यम् । यथोक्तं भाष्यकृता—

अतीन्द्रियार्थप्रतिपादकानि सूत्राण्यमून्यार्यभटोदितानि ।
तेषामशक्यार्थशतांशकोऽपि वक्तुं कुतोऽस्मत्सदृशैरशेषम् ॥ इति ।

Colophon : श्रीकर्णाटवंशकलशाम्बुधिकौस्तुभेन श्रीमद्गारुडशिखरशिरोमणिनरहरि-चरणारविंदभृङ्गराजेन श्रियःपतिदेशिककरुणाकटाक्षलब्धविद्याविशेषेण श्रीरघुनाथराजेन कृता श्रीमदार्यभटीयगोलपादव्याख्या सम्पूर्णा ।

From the opening stanzas of the commentary we learn that Raghunātha-rāja belonged to Karṇāṭa (Karnatak or Mysore) and was a king. His mother's name was Lakṣmī, and his genealogy was as follows :

Veṅkaṭa
|
Nagarāja
|
Koṇḍabhūpa
|
Raghunātha-rāja

The following stanza occurring in the commentary[1] throws light on the place where the commentator (Raghunātha-rāja) lived and wrote the commentary :

ब्रह्मेन्द्रादिकिरीटरत्नपटलश्रीपाटलाङ्घ्रिद्वयी-
पाथोजस्य रमानृसिंहवपुषा वासः परब्रह्मणः ।
ज्ञेया श्रीमदहोबिलेऽत्र विषुवच्छाया गुणः खत्रय-
स्तत्रैवाकृतिमानयोजनततिः भूमध्यरेखावधेः ॥

The last two lines mean :

> "Here at Ahobila, which is the abode of Parabrahma in the form of Ramānṛsiṁha, the equinoctial midday shadow (of a gnomon of 12 *aṅgulas*) is to be known as 3 (*aṅgulas*) and 30 (*vyaṅgulas*) ; also for the very same place the distance from the (Hindu) prime meridian is equal to 22 *yojanas*."[2]

This equinoctial midday shadow corresponds to latitude 15° 50′ approx. This conclusion is corroborated by the following facts :

(i) In an example set in the commentary,[3] the commentator gives the Rsine of the local latitude as equal to 962′ 38″.

(ii) At another place in the commentary,[4] the commentator gives the times of rising of the signs for his local place as follows :

Sign	Time of rising in *vinādikās*
Aries	243
Taurus	271
Gemini	311
Cancer	335
Leo	327
Virgo	313

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1. under *Ā*, iii. 6 (*c-d*).

2. This translation agrees with Raghunātha-rāja's interpretation. One *vyaṅgula* is one-sixtieth of an *aṅgula*.

3. under *Ā*, iv. 26.

4. Comm. on *Ā*, iv. 27.

In the commentary on verse 6(c-d) of the *Kālakriyā-pāda*, the local circumference of the Earth at Ahobila is given to be 3162½ *yojanas*, so that the distance 22 *yojanas* of that place from the (Hindu) prime meridian corresponds to 1° 40′.[1] Further, in an example solved in the commentary under verse 10 of the *Kālakriyā-pāda*, the longitude correction for Ahobila has been applied negatively. It follows that Ahobila lay 1° 40′ towards the east of the Hindu prime meridian.

From what has been said above, we find that Ahobila, the native place of the commentator Raghunātha-rāja, was situated approximately in latitude 15° 50′ N and longitude 1° 40′ E of the Hindu prime meridian. According to the verse quoted above, it was the seat of the Lakṣmī-nṛsiṁha temple. So, it is the same Ahobila as is situated in Kurnool district, Andhra State.[2]

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1. It should be noted that the number 3162½ *yojanas* accords to the *yojana*-reckoning of Āryabhata I, whereas the number 22 *yojanas* accords to the *yojana*-reckoning of the *Sūrya-siddhānta*. The latter number, when reduced to the *yojana*-reckoning of Āryabhaṭa I, would become 14⅗ *yojanas*.

2. Ahobila is a village and temple in the Kurnool District of Andhra State, situated in latitude 15° 8′ N and longitude 78° 45′ E on the Nallamalais. It is about 34 miles from Nandyal railway station, 22 miles from Nandyal to Allagadda by road and 12 miles from Allagadda to Ahobila by cart or on foot.

The temple at Ahobila is the most sacred Vaiṣṇava shrine in the District. It has three parts, namely : Diguva (lower) Ahobila temple at the foot of the hills, Yeguva (upper) Ahobila temple about four miles higher up, and a small shrine on the summit. The first is the most interesting as it contains beautiful reliefs of scenes from the *Rāmāyaṇa* on its walls and on the two great stone porches which stand in front of it, supported by pillars 8 feet in circumference, hewn out of the rock.

It is said that in ancient times Ahobila was the capital of the demon king Hiraṇyakaśipu, whose son Prahlāda was saved from the wrath of his father by God Nṛsiṁha at this very place.

There are three hills at Ahobila, *viz.*, *Garuḍādri*, *Vedādri* and *Acalacchāyā-meru*. The commentator Raghunātha-rāja has remembered God Nṛsiṁha, Lord of Garuḍāri, at the commencement of his commentary and has sought His protection.

Ahobila, as stated above, is now in Andhra State, but it appears from the commentary that in the time of Raghunātha-rāja it formed part of Karnataka of which he was the king.

In the commentary on verse 6(c-d) of the *Kalakriyā-pāda*, 1519 is mentioned as the current Śaka year. This corresponds to A.D. 1597 and is evidently the time of writing the commentary. The same year is mentioned at other places[1] in the commentary also.

The present commentary is based on those of Bhāskara I and Sūryadeva. It goes deep into explanatory details and is, on the whole, a very valuable work. The number of quotations from anterior works is large but the commentator refers only to a few of them by name. Amongst these may be mentioned the names of the *Brahma-siddhānta*, the *Soma-siddhānta*, the *Sūrya-siddhānta*, the *Pañca-siddhāntikā*, the *Bhāskara-bhāṣya* (*i e.*, Bhāskara I's commentary on the *Āryabhaṭīya*), the *Laghu-Bhāskarīya*, the *Triśatikā*, the *Utpala-parimala*, the *Siddhānta-śekhara*, the *Līlāvatī*, the *Siddhānta-śiromaṇi*, and Mallikārjuna Sūri's commentary on the *Śiṣya-dhī-vṛddhida* of Lalla. Amongst the authors mentioned are Vṛddha-Garga, Garga, Parāśara, Vīrabhadra, Vasiṣṭha, Devala, Siṁharāja, Varāhamihira, Brahmagupta, Lallācārya, and Mañjulācārya. Bhāskara I has been called Bhāṣyakāra ('author of the *Bhāṣya*').

The commentary contains a large number of solved examples. Forty-one of these examples have been taken from the commentary of Bhāskara I, fifteen from the commentary of Sūryadeva, and some from the works of Bhāskara II.

9. Commentary of Mādhava, son of Virūpākṣa

From the following passage occurring in the beginning of Mādhava's commentary on the *Bṛhajjātaka* of Varāhamihira, we learn that a commentary on the *Āryabhaṭīya*, giving rationales of the rules and illustrative examples, was also written by him :

आत्रेयगोत्रजो धीमान् विरूपाक्षार्यनन्दनः ।
माधवज्यौतिषाख्योऽहं वन्तुलान्वयसम्भवः ॥
आदावार्यभटीयस्य सिद्धान्तस्य सवासनाम् ।
उदाहरणसंयुक्तां टीकां कृत्वा ततः परम् ॥

नारदेन कृतायाश्च संहितायाः महर्षिणा ।
टीकां कृत्वा च संहृष्य बोधाय तदनन्तरम् ॥

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1. See comm. on $\bar{A}$, iii. 10, iv. 4, and iv. 5.

वराहमिहिरप्रोक्ते होरास्कन्धद्वयेऽपि च ।
लघुजातकहोराया विवृतिः प्रथमं कृता ॥
बृहज्जातकहोरायाष्टीकाश्चान्या निरीक्ष्य च ।
टीकां बालप्रबोधाय करिष्याम्यान्ध्रभाषया ॥ etc.[1]

The above passage shows that—

(i) Mādhava, the commentator, was a Brāhmaṇa of Ātreya Gotra, and belonged to the family (*anvaya*) of Vantula.

(ii) His father's name was Virūpākṣa.

(iii) His commentary on the *Āryabhaṭīya* was his earliest work.

(iv) His other commentaries were on the *Nārada-saṁhitā* and on the *Laghu-jātaka* and the *Bṛhajjātaka* of Varāhamihira.

As the commentator himself says in the last two lines of the above passage, his commentary on the *Bṛhajjātaka* was in Telugu. This shows that he belonged to the Andhra State. It is not known whether his commentary on the *Āryabhaṭīya* was composed in Sanskrit or in Telugu.

10. Bhūtiviṣṇu's commentary

A manuscript of Bhūtiviṣṇu's commentary on the *Āryabhaṭīya*, entitled *Bhaṭapradīpa*, exists in the Royal Library at Berlin.[2] The concluding verse of the commentary on the *Gītikā-pāda* (as reconstructed from the corrupt reading in the manuscript) runs as follows :—

भटेन पूर्वं दशगीतिसूत्रमतीव गूढार्थमुदाहृतं यत् ।
गुरुप्रसादादधिगम्य विद्वांस्तद् भूतिविष्णुः समवोचदित्थम् ॥

Bhūtiviṣṇu is the author of a commentary on the *Sūrya-siddhānta* also, of which an incomplete manuscript (containing a few pages in

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1. *A Descriptive Catalogue of the Sanskrit Manuscripts in the Government Oriental Manuscripts Library, Madras*, 1918, Vol. 24, Ms. No. 13835.

2. Catalogue No. 834, *De Handschriften-verzeichnisse der Koniglichen Bibliothek*, Erster Band, *Verzeichniss der Sanskrit-Handschriften*, by Weber, Berlin, 1853, p. 232.

There is also another manuscript of this commentary in the Anup Sanskrit Library, Bikaner. See Cat. No. 4447.

the beginning) exists in the Lucknow University Library, Lucknow.[1] It begins with :

भगवति कुरुत दृढतरां भक्ति प्रह्वप्रसादकृतदीक्षे ।
करिशिखरिकृतनिवासे देवे श्रीवल्लभे तस्मिन् ॥
गर्गान्वयशिरोरत्नं देवराजो महामतिः ।
भूतिविष्णुसुतः श्रीमानभूद् भूलोकभूषणम् ॥
भूतिविष्णुरभूत् पुत्रो ज्येष्ठस्तस्य सतां मतः ।
प्रतिभानवतां श्रेष्ठः श्रीमच्छब्देन सत्कृतः ॥

व्याचिख्यासा वर्तते सूर्यलब्धे
सिद्धान्तेऽस्मिस्तस्य मे भूतिविष्णोः ।
हस्तिक्ष्माभृन्मौलिरत्नं रमेश-
स्तत्साफल्यं कल्पयेदल्पबुद्धेः ॥

अत्र सिद्धान्तलक्षणं श्रीपतिराह—शतानन्दश्चास्ति etc.

"O Goddess—dedicated to bestowing favour on humble devotees, enable me to have firmer devotion towards God Viṣṇu who resides at Kari-śikhari (*i.e.*, the Elephant Mountain or Kāñcī).

"There was a learned and virtuous Devarāja, the crest-jewel of the lineage of Garga, the son of Bhūtiviṣṇu and an ornament of the terrestrial world. His eldest son, Bhūtiviṣṇu, was regarded as the best amongst the wise and intelligent and was honoured with the epithet 'Śrīmat'. That Bhūtiviṣṇu, who I am, has the desire to comment on the *Siddhānta* acquired from Sūrya (*i.e.*, *Sūrya-siddhānta*). May God Viṣṇu, the crest-jewel of the Hasti-giri (*i.e.*, the Elephant Mountain), accomplish that desire of this ignorant self."

The above passage shows that Bhūtiviṣṇu belonged to the lineage of Garga and that he was the son of Devarāja and the grandson of his own namesake Bhūtiviṣṇu. This Devarāja was a different person from Devarāja, the author of the *Kuṭṭākāra-śiromaṇi*, as the former belonged to the lineage of Garga and the latter to the lineage of Atri.

Bhūtiviṣṇu's commentary on the *Āryabhaṭīya* was written earlier than his commentary on the *Sūrya-siddhānta*, which is evident from the

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1. Accession No. 47070.

reference of the former in the latter. For, commenting on *SuSi*, i. 11-12, Bhūtiviṣṇu says :

तथा च भटप्रदीपे अस्माभिरुक्तम्—

'एवं हि तत्तद्ग्रहसम्भवानि स्युः सावनान्यत्र तु वासराणि ।' इति ।

The initial few pages of Bhūtiviṣṇu's commentary on the *Sūrya-siddhānta*, which are available in the Lucknow University Library, Lucknow, do not throw light on the time and place of Bhūtiviṣṇu. But they do contain numerous references to Śrīpati and quotations from his *Siddhānta-śekhara*, which shows that he lived posterior to Śrīpati (A.D. 1039). Similarly, his devotion to God Viṣṇu residing on the Elephant Mountain (*i.e.*, Kāñcī) suggests that he belonged to Tamilnadu, in South India.

11. Ghaṭīgopa's commentary

Two manuscripts of this commentary exist in the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum.[1]

This commentary begins thus :

गजाननं च वाणीं च श्रीसूर्यादीन् ग्रहानपि ।
पूर्वाचार्यांश्चार्यभटप्रमुखान् प्रणतोऽस्म्यहम् ॥

प्रणिपत्यैकमिति । कं ब्रह्माणम् । एकं कारणरूपतया, अनेकं कार्यरूपतया । वर्गाक्षराणीति । पञ्चविंशद्वर्गाक्षराणि क्रमादोजस्थानगतसंख्यावाचीनि । तत्र यकारो युग्म-स्थानगतत्रिसंख्यावाची । रेफादीनि सप्त चतुरादिसंख्यावाचीनि । वर्गावर्गाक्षराश्रिता अणादयो नव स्वरा उत्तरोत्तरस्थानवाचिनः । एवमष्टादशस्थानानां क्लृप्तिः ।

It ends thus ;

आर्यभटीयं नाम्ना इति । पूर्वं स्वयम्भुवा प्रणीतं यच्छास्त्रं तन्मूलं मया कृतमिदं मन्नाम्ना आर्यभटीयमिति ख्यातिं प्राप्नोति । सदा सत्यम् । महति कालान्तरे च विस्फुटार्थं भवति । अस्य प्रतिकञ्चुकं दोषाशंकित्वाद्विरोधं यः कुरुते स आत्मनः सुकृतायुषोः प्रणाशमेव करोति । अनेन यो भक्तिपुरस्सरमस्य शास्त्रस्य श्रवणादीनि करोति स चिरकालं सुकृतायुषी लब्ध्वा ऐहिकामुष्मिकफलभोगीत्युक्तं भवति । एवं पञ्चाशत्सूत्रम् ।

परमेश्वरपादाब्जपांसुपावितमूर्तिना ।
एतदार्यभटीयस्य यत्किञ्चिद् व्याहृतं मया ॥

1. Mss. Nos. 13305-A and T. 736,

गृह्णन्त्वपायविम्बाः करुणाविलचेतसः ।
सन्तो विहाय दोषौघं गुणलेशादृताशयाः ॥
घटीगोपाभिधानस्य वाङ्मनःकायवृत्तिभिः ।
यत्कृतं पद्मनाभस्य पूजा तदखिलं भवेत् ॥
इति गोलपादः समाप्तः ।

The closing verses of the commentary show that Ghaṭīgopa was a devotee of God Padmanābha and a pupil of Parameśvara. This Parameśvara, however, was different from his namesake, the author of the *Dṛggaṇita* (A.D. 1431), for, according to K.V. Sarma, Ghaṭīgopa, in his smaller Malayalam commentary on the *Āryabhaṭīya* (see below), quotes from the *Pañcabodha* of Putumana Somayāji, and Putumana Somayāji, according to K.V. Sarma, lived between A.D. 1675 and A.D. 1750, *i.e.*, more than two centuries later than Parameśvara, author of the *Dṛggaṇita*.

K.V. Sarma is of the opinion that this Ghaṭīgopa is the same person as Prince Godavarma Koyittampurān (A.D. 1810-60), a member of the scholarly family of Kilamanoor and a resident of Trivandrum, who bore the appellation 'Maṇikkāran' (=clockman) (in Malayalam), equivalent to 'Ghaṭīgopa' (in Sanskrit).

12. Kodaṇḍarāma's (A.D. 1807-83) commentary

A complete manuscript of Kodaṇḍarāma's commentary on the *kālakriyā-pāda* of the *Āryabhaṭīya*, in Sanskrit verses along with Telugu meaning, exists in the Government Oriental Manuscripts Library, Madras.[1] It is called *Āryabhaṭatantra-gaṇita*.

Kodaṇḍarāma is also the author of a work called *Āryabhaṭa-vāṇī*, which was meant to be a sequel to the *Āryabhaṭīya*. S. Kuppuswami Śāstrī describes this work as follows :

> "A treatise in stanzas of the *āryā* metre dealing with the Advaita-Vedānta. By Kodaṇḍarāma of Koṭikulapuḍi family. He states that he adds a fourth *Pāda* to the three *Pādas* of Āryabhaṭa, *viz.*, *Gaṇita*, *Kala* and *Gola*, wherein everything relating to calculation is explained, and that in the fourth

1. Ms. No. R. 371 (0).

Pāda called *Ānanda-pāda*, the nature of the Supreme Brahman is explained."[1]

(b) *Commentaries in Telugu*

13. Kodaṇḍarāma's commentary

Kodaṇḍarāma wrote a commentary in Telugu also. It is on the first three *Pādas* (*viz.*, *Gītikā*, *Gaṇita* and *Kālakriyā*) only, and bears the name *Sudhātaraṅga*.

This commentary has been edited by V. Lakshminarayana Sastri and published in *Madras Government Oriental Series* (No. CXXXIX) in 1956.

14. Virūpākṣa's commentary

A manuscript of this commentary exists in the Oriental Manuscripts Library, Mysore.[2]

(c) *Commentaries in Malayalam*

15. Kṛṣṇadāsa's commentary

A manuscript of Kṛṣṇadāsa's commentary covering the *Gītikāpada* occurs in the collection of K.V. Sarma. The beginning and end of it run as follows :

Beginning : श्रीगुरुभ्यो नमस्तेभ्यो दयालुभ्यो मुहुर्मुहुः ।
येषां प्रसादाच्छिष्यस्तु याति मन्दोऽप्यमन्दताम् ॥
व्यत्यस्तपादकमलं साचीकृतमुखाम्बुजम् ।
प्रपूरयन्तं मुरलीं नमामि पुरुषोत्तमम् ॥

वेदाङ्गङ्ङळिल् वच्चु प्रधानभूतमायुं स्कन्धत्रयात्मकमायुं इरिक्कुन्न ज्योतिश्शास्त्रत्तिन्टे मून्नु स्कन्धङ्ङळिल् वच्चु etc.

End : अतु कोण्टु ई शास्त्रत्तिन्टे अध्ययनं मोक्षोपयोगियाग्नि एन्नु सिद्धमायि ।

Colophon : दशगीतिसूत्रभाषा रचितेयं कृष्णदेवदासेन ।
मुरलीधरगोपालप्रीत्यै भूयाच्छुभं भवतु ॥
इति दशगीतिसूत्रभाषा समाप्ता ।

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1. *Triennial Catalogue of the Govt. Oriental Manuscripts Library, Madras*, Vol. III, Part I, Sanskrit A, Madras, 1922. Ms. No. R. 2156 (a).

2. Ms. No. B. 573.

Kṛṣṇadāsa is identified with Koccu-Kṛṣṇan Āśān (A.D. 1756-1812), of the family of Neṭumpayil in the Tiruvalla taluk of South Kerala, well known in Malayalam literary circles as the author of several poetical works. He is also the author of a number of astrological works in Malayalam.[1]

16. Kṛṣṇa's commentary

A manuscript of *Āryabhaṭīya-vyākhyā*, a commentary in Malayalam, entitled *Bhāṣāyāṁ Kṛṣṇa-ṭīkā*, exists in the library of the India Office, London.[2] It begins with the words :

आर्यभटाचार्यन् तन्त्रत्ते आरम्भिप्पान्

More details regarding this commentary are not known and it is difficult to say whether the author of this commentary was the same person as Kṛṣṇadāsa or different from him.

17-18. Two commentaries by Ghaṭīgopa

In addition to his commentary in Sanskrit (already noticed), Ghaṭīgopa wrote two commentaries in Malayalam, both on the *Gaṇita*, *Kālakriyā* and *Gola Pādas* only. The larger commentary extends to 1850 *granthas* (1 *grantha* = 32 letters), and the smaller one to 1200 *granthas*.

Of the larger commentary, there exist two manuscripts (Nos. C. 2333-A and T. 157 B) in the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum. The beginning and end of it are as follows :

Beginning : नटे आर्यभटाचार्यन् दशगीतिसूत्रमायिरिकुन्न प्रबन्धं कोण्टु अतीन्द्रियमायिरिक्कुन्न अर्थजातत्ते उपदेशिच्चिट्टु तन्मूलन्यायसिद्धमाकुन्न गणितपादत्तेयुं कालक्रियापादत्तेयुं गोलपादत्तेयुं चेय्युवानायिट्ट इष्टदेवतानमस्कारं चेय्तु शास्त्रत्ते आरम्भिक्कुन्नु—ब्रह्मकुशशि etc.

End : अज्ञानाद्वा यदपि मया ग्रन्थविस्तारभीते-
राचार्यार्याविवरणमिदं कल्पितं स्वल्पमेव ।
विद्वांसो ये विपुलमतयः क्षन्तुमर्हन्ति तत्त-
च्चापल्योक्तीरिव गुरुजनाः सानुरागाः शिशूनाम् ॥

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1. For details, see K. V. Sarma, *A history of the Kerala school of Hindu astronomy*, pp. 74-75.

2. Ms. No. 6273.

घटीगोपाभिधानस्य वाङ्मनःकायवृत्तिभिः ।
यत्कृतं पद्मनाभस्य पूजा तदखिलं भवेत् ॥

गोलपादभाषा समाप्ता ।

Of the smaller commentary, there are three manuscripts (Nos. 11014, L. 1334 and T. 157-A) in the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum, and one (No. 542-B) in the Government Sanskrit College Library, Tripunithura. This commentary begins thus :

गणितपादत्तिङ्कल् मुप्पत्तिमून्नार्यंकळेक्कोण्टु युक्तिसिद्धमायिरिक्कुन्न गणितत्ते चोल्लु-वान् तुटङ्ङुन्न आर्यभटाचार्यन् नटेत्ते सूत्रं कोण्टु इष्टदेवतानमस्कारवुं चिकीर्षितप्रतिज्ञयुं चेय्युन्नु – ब्रह्मकुशशि etc.

The commentary on the last two verses is in Sanskrit and not in Malayalam. It ends with the following sentence :

अनेन यो भक्तिपुरस्सरमस्य शास्त्रस्य अध्ययनश्रवणादीनि करोति स चिरकालं सुकृतायुषी लब्ध्वा ऐहिकामुष्मिकफलभोगी भवतीत्युक्तम् ।

which is exactly the same as in his Sanskrit commentary.

The colophon to *Gaṇita-pāda* runs thus :

आर्यभटीयस्यास्मिन् पादे गणिताभिधेऽतिगम्भीरे ।
यदिह घटीगोपेन प्रोक्तं किञ्चन बुधाः क्षमध्वमिदम् ॥

गणितपादभाषा समाप्ता ।

The colophon to *Gola-pāda* runs thus :

परमेश्वरपादाब्जपांसुपावितमूर्तिना ।
एतदार्यभटीयस्य यत् किञ्चिद् व्याहृतं मया ॥
गृह्णन्त्वगाधधिषणाः करुणाविलचेतसः ।
सन्तो विहाय दोषौघं गुणलेशावृताशयाः ॥
घटीगोपाभिधानस्य वाङ्मनःकायवृत्तिभिः ।
यत्कृतं पद्मनाभस्य पूजा तदखिलं भवेत् ॥

इति गोलपादभाषा समाप्ता ।

It is noteworthy that some of the verses occurring towards the end of all the three commentaries written by Ghaṭīgopa are exactly

the same and prove beyond doubt the common authorship of the three commentaries.

(d) *Commentary in Marathi*

19. Anonymous commentary in Marathi

A commentary (rather a translation) in Marathi exists in the Bombay University Library, Bombay.[1] The name of the author is not mentioned.

6.7 Works based on the *Āryabhaṭīya*

Of the works written on the basis of the *Āryabhaṭīya*, mention may be made of the following :

1. The works of Bhāskara I

For details see Introduction to Part II of this Series, pp. xxx ff.

2. The *Karaṇa-ratna* of Deva (A.D. 689) son of Gojanma

A manuscript of this work exists in the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum.[2]

The *Karaṇa-ratna* is a calendrical work in eight chapters, containing in all 183 verses. In the second opening stanza, the author says :

> "Having taken a deep plunge into the entire ocean of the *Āryabhaṭa-śāstra* with the boat of intellect, I have acquired this jewel, the *Karaṇa-ratna*, adorned by the rays of all the planets."[3]

This work, though essentially based on the teachings of the *Āryabhaṭīya*, is highly influeneed by the *Khaṇḍa-khādyaka* of Brahmagupta. It adopts a number of verses from the *Laghu-Bhāskarīya* and the *Khaṇḍa-khādyaka*.

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1. Ms. No. 334.
2. No. T.559.
3. आर्यभटशास्त्रजलधिं मतिनावा दूरमखिलमवगाह्य ।
   लब्धं मयेदमखिलग्रहांशुजटिलं करणरत्नम् ॥

This is the earliest work of the Āryabhaṭa school that states the precession of the equinoxes and the so-called *Śakābda*, *Manuyuga* and *Kalpa* corrections.

3. The *Graha-cāra-nibandhana* of Haridatta (or Haradatta)

This calendrical work was edited by K.V. Sarma and published by the Kuppuswami Sastri Research Institute, Mylapore, Madras, in 1954.

The work is in three chapters and states simplified rules and tables for finding the true longitudes of the planets and therefrom the *nakṣatra* and *tithi*, two of the five elements of the Hindu calendar.

This work does not prescribe any *bīja* correction to the mean longitudes of the planets, although it is conjectured that Haridatta was the author of the so-called *Śakābda* correction.

4. The *Śiṣya-dhī-vṛddhida* of Lalla (or Ralla)

The text of this work was published by Sudhakara Dvivedi at Benaras in A.D. 1886. In the opening stanzas, the author explains the scope of the work as follows :

> "That science of astronomy which, as told by Āryabhaṭa, is difficult to comprehend is being set forth by Lalla in such a way as to be easily understood by students.
>
> "Although having mastered the *śāstra* composed by Āryabhaṭa, his pupils (or followers) have written astronomical *tantras*, but they have not been able to describe the methods properly. I shall, therefore, state the procedures stated by him in proper sequence."[1]

In the penultimate stanza of the *Grahagaṇita* part of the same work, he again says :

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1. आचार्यार्यभटोदितं सुविषमं व्योमौकसां कर्म य-
च्छिष्याणामभिधीयते तदधुना लल्लेन धीवृद्धिदम् ॥
विज्ञाय शास्त्रमलमार्यभटप्रणीतं
तन्त्राणि यद्यपि कृतानि तदीयशिष्यैः ।
कर्मक्रमो न खलु सम्यगुदीरितस्तैः
कर्म ब्रवीम्यहमतः क्रमशस्तदुक्तम् ॥

> "Lalla ... has composed this *tantra* which yields the same results as the *Āryabhaṭa-siddhānta* (*i.e.*, the *Āryabhaṭīya*)."

Regarding his parentage, the author (Lalla) himself writes :

> आसीदशेषबुधवन्दितपादपद्मस्तालध्वजो गुणवदग्रजवंशजन्मा ।
> साम्बस्ततोऽजनि जनेक्षणकैरवेन्दुर्भट्टस्त्रिविक्रम इति प्रथितः पृथिव्याम् ॥
> लल्लेन तस्य तनयेन शशांकमौलेः शैलाधिराजतनयादयितस्य शम्भोः ।
> सम्पूज्य पादयुगमार्यभटाभिधानसिद्धान्ततुल्यफलमेतदकारि तन्त्रम् ॥

This shows that he was a son of Sāmba, popularly known as Bhaṭṭa Trivikrama, and a grandson of the learned scholar Tāladhvaja, and that he was a Brāhmaṇa.

Chronologically, Lalla comes after Bhāskara I and Brahmagupta, but in the absence of any definite evidence his date could not be fixed so far. On the basis of a passage (*śake nakhābdhirahite* etc.) generally ascribed to him, it is conjectured that he lived about A.D. 748.[1] But this date is doubtful, because the said passage does not lead to any definite conclusion. There is, however, no doubt that Lalla lived sometime between A.D. 665 and A.D. 904. The former is the date of the *Khaṇḍa-khādyaka* on which Lalla wrote a commentary, and the latter the date of Vaṭeśvara who has utilized the *Śiṣya-dhī-vṛddhida* of Lalla in writing his *Siddhānta.*

Lalla's places of birth and activity are also unknown. But the following example, which is the only example of this kind occurring in the *Śiṣya-dhī-vṛddhida*,[2] probably refers to the place where he lived :

> "Knowing that the sum of the Rsines of the latitude and the colatitude is 1308′ and that the difference of the same is 538′, say what are the Rsines of the latitude and the colatitude *here.*"[3]

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1. See, for example, introduction to P. C. Sengupta's English translation of the *Khaṇḍa-khādyaka*, pp. xxvi-xxvii.

2. II, xii. 22.

3. लम्बाक्षज्यायोगं वस्वम्बरपावकेन्दवो दृष्ट्वा ।
   वसुगुणविषया विवरं लम्बाक्षज्ये कियत् स्तोऽत्र ॥

If x and y denote the two quantities, then

$$x = 923' = \text{Rsin } (15° \, 34') \text{ approx.}$$

$$y = 385' = \text{Rsin } (6° \, 26') \text{ approx.}$$

Thus the latitude of the place referred to in the above example is either 15° 34′ or 6° 26′. The latter alternative is impossible as the circle of latitude 6° 26′ does not cross the Indian continent. So we infer that Lalla lived in latitude 15° 34′ N.

There are also reasons to believe that Lalla belonged to Lāṭadeśa (*i.e.*, Gujarat). For, in his *Śiṣya-dhī-vṛddhida,* Lalla has made a special reference to the ladies of the Lāṭa country. He has compared the half-phased Moon with the forehead of a lady of the Lāṭa country (*lāṭī*). Although the *Śiṣya-dhī-vṛddhida* claims to set forth the teachings of the *Āryabhaṭīya,* the impact of the teachings of Brahmagupta on this work is also visible. Two features of the *Śiṣya-dhī-vṛddhida* deserve special notice : (i) arrangement of subject matter under two distinct heads—*Grahagaṇita* (dealing with astronomical calculations) and *Golādhyāya* (dealing with the celestial sphere, cosmogony, astronomical instruments, etc.), and (ii) language. This arrangement has been followed by Vaṭeśvara (A.D. 904) in his *Siddhānta,* by Bhāskara II (A.D. 1150) in his *Siddhānta-śiromaṇi,* by Jñānarāja (A.D. 1503) in his *Siddhānta-sundara,* and other later writers. The language used by Lalla is, at places, highly poetic and appealing. Some of his expressions and similies are so nice that posterior writers could not resist copying them. One can easily find a number of passages in the works of Vaṭeśvara, Śrīpati and Bhāskara II which have been copied from the *Śiṣya-dhī-vṛddhida* of Lalla.

5. The *Karaṇa-prakāśa* of Brahmadeva (A.D. 1092)

This calendrical work was edited by Sudhākara Dvivedī together with his own commentary. The epoch used in this work is A.D. 1092.

This work holds an important place amongst the calendrical works. It makes use of the *bīja* correction prescribed by Lalla, and *tithis* calculated from this work differ by about 2 to 3 *ghaṭīs,* being in excess, from those calculated from the parameters of the *Āryabhaṭīya.* This work was in use in South India, particularly in Mahārāṣṭra,

amongst Vaiṣṇavas, who preferred the 11th *tithi* calculated from this work. For details, see Dīkṣita's *Bhāratīya-Jyotiṣa-śāstra* (Marathi), pp. 240-42.

6. The *Bhaṭatulya* of Dāmodara

The epoch used in this work is A.D. 1417. The author Dāmodara was a son of Padmanābha (*c.* A.D. 1400) and a grandson of Nārmada (*c.* A.D. 1375). Use of Lalla's *bīja* correction is made in this work also. A manuscript of this work exists in the Deccan College Library, Poona. The second stanza therein runs as follows :

> "I, Dāmodara, bowing to the lotus-like feet of my teacher Padmanābha, write, for the pleasure of the learned, this work, which will yield the same results as those of Āryabhaṭa, by making use of the *pratyabda-śuddhi* method."[1]

For details see Dīkṣita, *ibid.*, pp. 354-56.

7. The *Karaṇa-paddhati* of Putumana Somayāji (A.D. 1732)

This work has been published in *Trivandrum Sanskrit Series* (No. 126), and the *Madras Government Oriental Series* (No. 98). The latter contains two Malayalam commentaries also.

8. The *Āryabhaṭa-siddhānta-tulya-karaṇa* by Vīrasiṁhagaṇaka son of Kaśirāja

Three manuscripts of this work occur in Anup Sanskrit Library, Bikaner.[2]

6.8. Transmission to Arab

The *Āryabhaṭīya* was taken to Arab where it was translated into Arabic by Abul Hasan Ahwazī under the title *Āryabhaṭa* (misread as *Arajbahara* or *Arajbahaz*). The Arabians misunderstood the exact significance of the title of the work and wrongly thought that it meant 'one thousandth part'.[3]

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1. दामोदरः श्रीगुरुपद्मनाभपदारविन्दं शिरसा प्रणम्य ।
प्रत्यब्दशुद्धयाऽऽर्यभटस्य तुल्यं विदां मुदेऽहं करणं करोमि ॥

2. Mss. Nos. 4448, 4449 and 4450.

3. See *Arab aur Bhārat ke sambandha*, by Maulana Saiyad Sulaiman Nadavī, translated into Hindi by Ram Chandra Varma, pub. by Hindustani Academy, Allahabad, 1930, p. 113.

7. THE ĀRYABHAṬA-SIDDHĀNTA

From the writings of Varāhamihira (died A.D. 587), Brahmagupta (A.D. 628), Bhāskara I (A.D. 629), Govinda-svāmī (ninth century), Mallikārjuna Sūri (A. D. 1178), Rāmakṛṣṇa Ārādhya (1472 A. D.), Maithila Caṇḍeśvara, Bhūdhara (A.D. 1572) and Tamma Yajvā (A.D. 1599), it is now established beyond doubt that Āryabhaṭa I, the author of the *Āryabhaṭīya*, wrote at least one more work on astronomy which was known as *Āryabhaṭa-siddhānta*. Unlike the *Āryabhaṭīya* in which the day was measured from one sunrise to the next, this work reckoned the day from one midnight to the next as was done in the *Sūrya-siddhānta*. The astronomical parameters and methods given in the *Āryabhaṭa-siddhānta* differed in some cases from those of the *Āryabhaṭīya*. The important differences between the two works have been noted by Bhāskara I in Chapter VII of his *Mahā-Bhāskarīya*. Some of the typical methods and the astronomical instruments described in the *Āryabhaṭa-siddhānta* have been mentioned by Mallikārjuna Sūri, Tamma Yajvā, Rāmakṛṣṇa Ārādhya and others in their commentaries on the *Sūrya-siddhānta*. The astronomical parameters and methods ascribed to the *Āryabhaṭa-siddhānta* are generally the same as those found in Varāhamihira's version of the *Sūrya-siddhānta* and the *Sumati-mahātantra* of Ācārya Sumati of Nepal, which was based on the *Sūrya-siddhānta*. It appears that the *Āryabhaṭa-siddhānta* was an independent work like the *Āryabhaṭīya* and that it bore the same relation to the earlier *Sūrya-siddhānta* as the *Āryabhaṭīya* bore to the earlier *Svāyambhuva-siddhānta*; and that the *Sūrya-siddhānta* summarized by Varāhamihira was the one anonymously revised by Lāṭadeva in the light of the *Āryabhaṭa-siddhānta*. This is, perhaps, the reason why both Āryabhaṭa I and Lāṭadeva are sometimes referred to as the authors of the *Sūrya-siddhānta*.

7·1. The *Āryabhaṭa-siddhanta* and the *Āryabhaṭīya*

The following tables exhibit the main differences between the astronomical parameters of the *Āryabhaṭīya* and the *Āryabhaṭa-siddhānta* according to Bhāskara I.

Table 1. Diameters and distances of planets in *yojanas*

	Āryabhaṭīya	*Āryabhaṭa-siddhānta*
Earth's diameter	1050	1600
Sun's diameter	4410	6480
Moon's diameter	315	480
Sun's distance	459585	689358
Moon's distance	34377	51566
$\frac{\text{circumference of the sky}}{\text{revolutions of the Moon}}$	216000	324000

The numbers in the second and third columns are in the ratio 2 : 3, approximately. This is due to the fact that the measures of *yojana* employed in the two works are in the ratio 3 : 2.

Table 2. Civil days, Omitted lunar days, and Revolutions of *Śīghrocca* of Mercury and Jupiter in a period of 43,20,000 years

	Āryabhaṭīya	*Āryabhaṭa-siddhānta*
Civil days	1,57,79,17,500	1,57,79,17,800
Omitted lunar days	2,50,82,580	2,50,82,280
Revolutions of *Śīghrocca* of Mercury	1,79,37,020	1,79,37,000
Revolutions of Jupiter	3,64,224	3,64,220

It may be pointed out that the difference of 300 days between the civil days of the two works was so adjusted that both the works indicated the same epoch at the end of the Kali year 3600 mentioned in the *Āryabhaṭīya*. Since 3600 years$=1{,}57{,}79{,}17{,}500/1200=13{,}14{,}931{\cdot}25$ days according to the *Āryabhaṭīya* and$=1577917800/1200=1314931{\cdot}50$ days according to the *Āryabhaṭa-siddhānta*, the Kali year 3600 ended exactly on Sunday, March 21, A D. 499, at mean noon at Laṅkā or Ujjayinī, according to both the works of Āryabhaṭa I.

Table 3. Longitudes of the planets' apogees (or aphelia) in 499 A.D.

Planet	*Āryabhaṭīya*	*Āryabhaṭa-siddhānta*
Sun	78°	80°
Mars	118°	110°
Mercury	210°	220°
Jupiter	180°	160°
Venus	90°	80°
Saturn	236°	240°

Table 4. Dimensions of planets' *manda* epicycles

	Āryabhaṭīya odd quadrant	*Āryabhaṭīya* even quadrant	*Āryabhaṭa-siddhānta*
Sun	13° 30′	13° 30′	14°
Moon	31° 30′	31° 30′	31°
Mars	63°	81°	70°
Mercury	31° 30′	22° 30′	28°
Jupiter	31° 30′	36°	32°
Venus	18°	9°	14°
Saturn	40° 30′	58° 30′	60°

The circumference of a planet's concentric or deferent (or mean orbit) is supposed to be of 360 units (called degrees) in length and the above dimensions are on the same scale.

Table 5. Dimensions of planets' *śīghra* epicycles

	Āryabhaṭīya odd quadrant	*Āryabhaṭīya* even quadrant	*Āryabhaṭa-siddhānta*
Mars	238° 30′	229° 30′	234°
Mercury	139° 30′	130° 30′	132°
Jupiter	72° 00′	67° 30′	72°
Venus	265° 30′	256° 30′	260°
Saturn	40° 30′	36° 00′	40°

7·2. The astronomical instruments and special methods of the *Āryabhaṭa-siddhānta*

Rāmakṛṣṇa Ārādhya (A.D. 1472) has quoted a set of 34 verses (composed in *anuṣṭubh* metre) from the chapter of the *Āryabhaṭa-siddhānta* dealing with the astronomical instruments. The instruments described in these verses are : (1) *Chāyā-yantra* ('the shadow instrument'), (2) *Dhanuryantra* ('the semi-circle'), (3) *Yaṣṭi-yantra,* (4) *Cakra-yantra* ('the whole circle'), (5) *Chatra-yantra* ('the umbrella'), (6) Water instruments, (7) *Ghaṭikā-yantra,* (8) *Kapāla-yantra,* and (9) the gnomon.[1] Of these instruments, some were indeed devised by Āryabhaṭa I. The

1. For details see K.S. Shukla, 'Āryabhaṭa I's astronomy with midnight day-reckoning', *Gaṇita,* Vol. 18, No. 1, pp. 83-105.

gnomon (as described by Āryabhaṭa I) and the water instruments have been generally attributed to him. It is in connection with these instruments that the commentators of the *Sūrya-siddhānta* have remembered him.

Mallikārjuna Sūri (A.D. 1178), Rāmakṛṣṇa Ārādhya (A.D. 1472) and Tamma Yajvā (A.D. 1599) have referred also to some special methods of the *Āryabhaṭa-siddhānta.* Of these methods, one relates to the approximate determination of time from the shadow of the gnomon. The method is interesting and also unique as it does not occur in any other known work on Indian astronomy. It may be briefly described as follows :

When the Sun is in Scorpio, Capricorn or Aquarius, and it is within 2 *ghaṭīs* from noon, set up a gnomon of 9 digits on the east-west line in such a way that the tip of its shadow may fall on the north-south line. Then the digits of the distance of the gnomon from the intersection of the east-west and north-south lines would approximately give the *ghaṭīs* to elapse before noon or elapsed since noon (according as the observation is made before noon or after noon).

7·3. Popularity of the *Āryabhaṭa-siddhānta* and the *Khaṇḍa-khādyaka* of Brahmagupta

The *Āryabhaṭa-siddhānta* was a popular work and was studied throughout India. It was mentioned in the sixth century by Varāhamihira of Kāpitthaka (near Ujjayinī), in the seventh century by Brahmagupta of Bhinmal (in Rajasthan) and Bhāskara I of Valabhī (in Kathiawar), in the ninth century by Govinda-svāmī of Kerala, in the twelfth century by Mallikārjuna-Sūrī of Āndhra and Maithila-Caṇḍeśvara of Banaras in Uttar Pradesh, in the fifteenth century by Rāmakṛṣṇa-Ārādhya of Āndhra, and in the sixteenth century by Bhūdhara of Kāmpilya (modern Kampil, twenty-eight miles north-east of Fatehpur in the Farrukhabad district, Uttar Pradesh) and Tamma-Yajvā of Āndhra.

There are reasons to believe that in the seventh century the popularity of this work in north India was at its highest peak and it was used not only as a text book of astronomy but also in everyday calculations such as those pertaining to marriage, nativity etc. The celebrated Brahmagupta who, in his youth, was a bitter critic of Āryabhaṭa I was so much impressed by its popularity that he

could not resist the temptation of bringing out an abridged edition of this work under an attractive title, 'Food prepared with sugarcandy' (*Khaṇḍa-khādyaka*). It was so much liked in some parts of India that it is in use even today.

Brahmagupta was not in complete agreement with the teachings of Āryabhaṭa I. So he planned his *Khaṇḍa-khādyaka* in two parts. In Part I he summarized the teachings of the *Āryabhaṭa-siddhānta* without making any alteration, modification or addition (but rectifying one or two rules whose inaccuracy was obvious to him); and in Part II he stated the corrections and modifications which had to be applied to Part I in order to get accurate results. In the opening stanzas of the two parts, Brahmagupta himself says :

(*Part I*) : "Having bowed in reverence to God Mahādeva, the cause of creation, maintenance and destruction of the world, I set forth the *Khaṇḍa-khādyaka* which yields the same results as the work of Āryabhaṭa.

"As it is generally not possible to perform calculations pertaining to marriage, nativity, and so on, every day by the work of Āryabhaṭa, hence this smaller work giving the same results."

(*Part II*) : "As the process of finding the true longitudes of the planets as given by Āryabhaṭa does not make them agree with observation, so I shall speak of this process (now)."

A sad consequence of the composition of the *Khaṇḍa-khādyaka* was that the original work of Āryabhaṭa I on which it was based was lost. The *Khaṇḍa-khādyaka,* however, received, wide acclamation and, though it was a calendrical work, a large number of commentaries were written on it. Amongst the commentators of this work were Balabhadra, whose commentary (*ṭīkā*) has been mentioned by Al-Bīrūnī (A.D. 973-1048); Pṛthūdaka (A.D. 860), whose commentary (*vivaraṇa*) has been edited by P. C. Sengupta; Lalla, whose commentary (*Khaṇḍa-khādyaka-paddhati*) has been mentioned by Āmarāja (*c.* A.D. 1200) ; Bhaṭṭotpala (A. D. 968), whose commentary (*vivṛti*) has been edited by Bina Chatterjee ; Varuṇa (*c.* A. D. 1040), whose commentary (*udāharaṇa*) is extant though not printed ; Someśvara, whose commentary has been mentioned by Āmarāja (*c.* A.D. 1200); Āmarāja (*c.* A.D. 1200),

whose commentary (*vāsanābhāṣya*) has been edited by Babuaji Misra; and Śrīdatta, a manuscript of whose commentary exists in Nepal. An anonymous commentary (*udāharaṇa*) exists in the India Office Library, London, and another written in Nepali in the Lucknow University Library, Lucknow. Sūryadeva (*b.* A.D. 1191) proposed to write a commentary on the *khaṇḍa-khādyaka*[1] but it is not known whether he actually wrote it.

The *Khaṇḍa-khādyaka* reached Arab where it was translated into Arabic under the title *Zīj-al-Arkand*, and was widely used.[2] It was retranslated into Arabic under the title *Az-Zīj Kandakātik al-Arabī* (=*The Arabic Khaṇḍa-khādyaka*) by the Persian scholar Al-Bīrūnī (A.D. 973-1048), who has quoted some of the methods of this work in his other works. (*E. g.*, see *Risā'il, II*, p. 150).

From Arab, the *Khaṇḍa-khādyaka* reached Europe and had its impact on astronomy there. O. Neugebauer has shown that "Kepler's theory of parallax is identical with the theory of the *Khaṇḍa-khādyaka*."[3]

8. THE PRESENT EDITION OF THE ĀRYABHAṬĪYA

(a) *Sanskrit Text*

The Sanskrit text incorporated in the present work is edited critically and all possible efforts have been made to reconstruct it as authentically as possible. It is based on : (i) original manuscripts of the text, (ii) available commentaries and (iii) quotations from later astronomers. The said three sources, it might be seen, are complementary and mutually corrective. Thus, while the text manuscripts present the text as handed down by manuscript tradition, the commentaries containing the meanings and derivations of the words in the text help in correcting scribal and other errors, besides indicating textual variants. Quotations in later works, cited either by way of

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1. Vide his statement towards the end of his commentary on Śrīpati's *Jātaka-paddhati*.

2. See E. S. Kennedy, *A survey of Islamic astronomical tables*, p. 138. According to Kennedy, it "was translated into Arabic, at or before the time of Yā'qūb ibn Ṭāriq, and was widely used".

3. See O. Neugebauer, *The astronomical tables of Al-Khwārizmī*, p. 124.

approbation in establishing a point or by way of refutation by a critic, are particularly helpful in deciding upon the correct readings of the text.

1. Text manuscripts[1]

Seven palmleaf manuscripts in Malayalam script, designated A to G (noticed below), have been collated towards fixing the text of the *Āryabhaṭīya*.

A. Ker[2]. 475-A. Mal. (Malayalam script), Pl. (palmleaf), Cm. (complete) ; 17 cm. × 5 cm., 7ff., 7 lines per page with about 36 letters per line. The verses have been written continuously through the entire length of folios on both the pages. Old, damaged and brittle to the touch. Inked and revised. The writing is quite readable but not shapely. The date of the ms. is given in a post-colophonic Kali chronogram in the *kaṭapayādi* notation which reads, *sevyo dugdhābdhitalpaḥ* (16,99,817), and corresponds to A.D. 1552.

The astronomical codex which contains A belonged originally to the reputed scholarly Nampūtiri family of Kūṭallūr in South Malabar, and carries the undermentioned works, all on mathematics and astronomy : A. *Āryabhaṭīya* of Āryabhaṭa ; B. *Mahā-Bhāskarīya* and C. *Laghu-Bhāskarīya* of Bhāskara I ; D. *Siddhānta-darpaṇa* and E. *Tāntra-saṅgraha* of Nīlakaṇṭha Somayājī ; F. *Līlāvatī* of Bhāskara II ; G. *Pañcabodha*, anon. ; H. *Laghumānasa* of Muñjāla ; I. *Candracchāyā-gaṇita* of Nīlakaṇṭha Somayājī; and J. *Goladīpikā* and K. *Grahaṇāṣṭaka*, both of Parameśvara (A.D. 1431).

B. Ker. 5131-B. Mal., Pl., Cm. ; 56 cm. × 5 cm., 4 ff., 8 lines per page with about 75 letters per line. Old, damaged and brittle. Inked and revised. Readable writing. Generally correct text. Neither dated nor scribe mentioned. The other work contained in this codex is *Bhāgavata-Purāṇa* numbered as 5131-A.

The codex has been procured from Shri Vāsudevan Nampūtiri of the village of Mārappaḍi in Central Kerala.

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1. The material of this section was supplied by K.V. Sarma.

2. Ker. stands for the Kerala University Or. Research Inst. and Mss. Library, Trivandrum.

C. Ker. 13300. Mal., Pl., Cm., 40 cm.×5 cm.; 7 ff. with 7 lines a page and about 50 letters a line. Old, brittle and damaged, some of the folios being torn. Neither dated, nor scribe mentioned. The text preserved is fairly accurate.

The works contained in this astronomical codex are: A. *Āryabhaṭīya*, B. *Sūrya-siddhānta*, C. *Laghumānasa* of Muñjāla, D. *Mahā-Bhāskarīya* of Bhāskara I, and E. *Dṛggaṇita* of Parameśvara.

The codex was procured by Shri Kāruvelil Nīlakaṇṭha Piḷḷai of Karthikappalli (S. Kerala) from an unidentified source.

D. Ker. 13305-B. Mal., Pl., Icm., 15 cm.×5 cm.; 12ff. with 8 lines a page and about 24 letters a line. Late ms., in good preservation. Very legible writing. Inked and revised, the reviser's corrections being identifiable by their not being inked. The text preserved is accurate. The ms. is not dated; neither is any scribe mentioned.

The codex contains also the *Āryabhaṭīya-vyākhyā* in Sanskrit by Ghaṭīgopa, which is catalogued as No. 13305-A. The codex belonged originally to the family collection of Paṭiññāreṭattu Pīṣāram in Kiṭaṅṅūr in Central Kerala.

E. Trip. 542-A, belonging to the Govt. Sanskrit College, Tripunithura, near Cochin. Mal., Pl., Cm., 20 cm.×3 cm.; 11 ff, with 10 lines a page and about 24 letters a line. A comparatively late manuscript, written in shapely script. The text preserved is generally correct. No date is given, nor is any scribe named.

The works contained in this astronomical codex are: A. *Āryabhaṭīya*, B. *Āryabhaṭīya-vyākhyā* in Malayalam by Ghaṭīgopa and C. *Veṇvāroha* by Mādhava with the Malayalam gloss of Acyuta Piṣāraṭi.

F. Ker. 501-A. Mal., Pl., Icm., 20 cm.×3 cm.; 12 ff., with 7 letters a page and about 25 letters a line. Old and damaged, with the corners worn out on account of constant use. Lacks the *Gītikāpāda*. Inked and revised, the corrections being uninked. The text preserved is accurate. No date or scribe has been mentioned.

The works contained in the codex are: A. *Āryabhaṭīya*, B. *Cāṭuślokāḥ (nānāgranthoddhṛtāḥ)*, and C. *Muhūrtapadavī*. The codex formed part of the famous mediaeval collection of the Deśamaṅgalam

Vāriyam in North Kerala, as known from an uninked marginal statement on the first folio, which reads : *Deśamaṅgalattu Vāriyatte Āryabhaṭādi.*

G. Ker. C. 2475-B. Mal., Pl., Icm., extending up to *Gaṇita*, verse 2 only, 25 cm. × 5 cm., 1 f., with 10 lines a page and about 37 letters a line. Carefully written in beautiful hand. Scrupulously revised. Not dated, nor any scribe mentioned. The text preserved is accurate.

The other work contained in the codex is the *Āryabhaṭīya-vyākhyā* by Sūryadeva Yajvā. The codex contains also folios with some miscellaneous matter inscribed thereon. The codex belonged originally to the library of the royal principality of Eḍappaḷḷi in Central Kerala.

All these manuscripts are completely independent of each other. Neither do they present any consistent common characteristic so as to enable them being grouped in any order or formulate any *stemma codicum* to portray their descent.

2. Text preserved in the commentaries

Of the commentaries on the *Āryabhaṭīya*, those by Bhāskara I (A.D. 629), Sūryadeva (*b.* A.D. 1191), Parameśvara (A.D. 1431) and Nīlakaṇṭha (A.D. 1500) are available in print. These commentaries have been referred to as *Bh.*, *Sū.*, *Pa.*, and *Nī.*, respectively, and the following editions have been used in the collation of the text :

Bh. Edited by K.S. Shukla in Part II of the present series.

Sū. Edited by K.V. Sarma in Part III of the present series.

Pa. Edited by H. Kern at Leiden in 1874.

Nī. Edited by K. Sāmbaśiva Śāstrī (*TSS*, Nos. 101 and 110) in 1930, 1931 and by Suranad Kunjan Pillai (*TSS*, No. 185) in 1957.

Commentaries by Someśvara, Yallaya (A.D. 1480), Raghunātha-rāja (A.D. 1597), Kṛṣṇadāsa (A.D. 1756-1812) and Ghaṭīgopa (*c.* A.D. 1800-60) are available in manuscript form. The following manuscripts (designated as *So.*, *Ya.*, *Ra.*, *Kṛ.*, and *Gh*, respectively) have beed used in the collation of the text :

So. Transcript. Accession No. 45886 of the Lucknow University Library, Lucknow. (Transcribed from Bs. No. 272, Catalogue No. 335, Accession No 2495 of the Bombay University Library, Bombay. The original manuscript is complete but extremely defective and full of inaccuracies and omissions).

Ya. Transcript in the collection of A. N. Singh. It contains *Gaṇita-pāda* (up to vs. 28), *Kālakriyā-pāda* and *Gola-pāda.*

Ra. Transcript. Accession Nos. 45771, 45772 and 45773 of the Lucknow University Library, Lucknow. Complete in four *Pādas.*

Kṛ. Transcript in the collection of K.V. Sarma. The original of this transcript, which contains the *Gītikā-pāda* only, is available in the Government Sanskrit College Library, Tripunithura, Kerala.

Gh. Transcript of the smaller version of Ghaṭīgopa's Malayalam commentary in the collection of K. V. Sarma. It is a copy from Ms. No. 542-B of the Government Sanskrit College Library, Tripunithura. See E. above.

The variant readings noted or discussed in the commentaries have also been generally taken into consideration.

3. Quotations from later astronomers

Extracts from the *Āryabhaṭīya* occur as quotations in the *Brāhma-sphuṭa-siddhānta* of Brahmagupta, Pṛthūdaka's (A.D. 860) commentary on the *Brāhma-sphuṭa-siddhānta,* Govinda-svāmī's commentary on the *Mahā-Bhāskarīya,* and Śaṅkaranārāyaṇa's (A. D. 869) and Udaya-divākara's (A.D. 1073) commentaries on the *Laghu-Bhāskarīya.* These works have been referred to as *Br.*, *Pṛ.*, *Go.*, *Śa.* and *Ud.* respectively, and the following editions or manuscripts of them have been used :

Br. Edited by Sudhākara Dvivedī, Benaras, 1902.

Pṛ. Photostat copy of Ms. Egg. 2769 : No. 1304 of the India Office Library, London. Belonging to the Lucknow University Library, Lucknow, Accession No. 47047.

Go. Edited by T.S. Kuppanna Sastri and published in *Madras Government Oriental Series*, No. CXXX, 1957.

Śa. Transcript in the collection of A.N. Singh. Complete.

Ud. Transcript. Accession No. 46338 of the Lucknow University Library, Lucknow. Complete.

4. Variations in reading

The collation of the manuscripts did not reveal many significant variations in the text. In the first *Pāda*, the variations are mostly phonetic :

(i) ख्वृ, ख्षृ (vs. 3) (ii) ञुष्खिघ, ञुंष्खिघ (vs. 4)

(iii) मनुयुगाः, मनुयुग (vs. 5) (iv) नवराषह्, नवरषाह्, नवरषहा (vs. 9)

(v) भा, भ (vs. 10) (vi) ग्ला, ग्ल (vs. 10)

(vii) किघ्व, किघ्वा (vs. 12)

Other variations in vs. 12 also seem to be inspired by phonetic requirements. This *Pāda* was generally learnt by heart and the students seem to have varied the readings to suit their pronunciations without affecting the meaning of the text.

One significant variation in reading in this *Pāda* is :

भं for भूः (vs. 6)

which seems to have been deliberately made under the pressure of Varāhamihira's criticism of the theory of the Earth's rotation.

Variations in *Pāda* II are generally verbal and due to the scribes. The following variations, though not significant, are noteworthy :

(i) भुजावर्गयुतः कोटीवर्गः कर्णः स एव । (Pṛthūdaka)]
यश्चैव भुजावर्गः कोटीवर्गः कर्णः स एव । (Others)] (vs. 17)

(ii) तन्मूलं मूलार्धोनं कालहृतं स्वमूलफलम् । (Bhāskara and Someśvara)]
मूलं मूलार्धोनं कालहृतं स्यात् स्वमूलफलम् । (Others)] (vs. 25)

In *Pāda* III, there is one significant variation in reading, *viz.* :

भावर्ताश्चापि for क्वावर्ताश्चापि (vs. 5)

This, too, seems to have been made when *bhūḥ* was changed into *bhaṁ*. Other noteworthy, though not very significant, variations are :

एवमेव for व्यत्ययेन

मन्देऽर्धं ऋणधनं भवति पूर्वम् for मन्दादर्धं ऋणधनं भवति पूर्वे

in vs. 22., both of these being mentioned by Bhāskara I.

In *Pāda* IV also, there is one significant reading-difference, *viz.* :

पञ्चदशांशे for तच्चतुरंशे (vs. 14)

which is inspired by the teaching of Brahmagupta. Other notable variations are :

ब्राह्मदिवस for ब्रह्मदिवस (vs. 8)

दिनतुल्ययैव रात्र्या or तत्तुल्ययैकराद्या for दिनतुल्ययैकरात्र्या (vs. 8)

यमकोट्यां for यवकोट्यां[1] (vs. 13)

अर्धं त्वपसव्यगतं (*Bh.*, *So.*), अपसव्यगतं तथार्धं (others) (vs. 16)

क्रमोत्क्रमतः for क्रमोत्क्रमशः (vs. 27)

भक्ता for भजिता (vs. 35)

छायाया दीर्घत्वं for भूच्छायादीर्घत्वं (vs. 39)

स्थित्यर्धमस्य मूलं for स्थित्यर्धं तन्मूलं (vs. 42)

स्थित्यर्धं for स्थितिमध्य (vs. 44)

देवता for ब्रह्मणः (vs. 49)

सद्यत् or सत्यं for नित्यं (vs. 50)

5. Selection of readings

In the selection of readings for the Sanskrit text, preference has been given to the most appropriate and, if possible, the oldest readings. Readings which were considered to be wrong or due to subsequent alteration in the text, or else, were less appropriate and unacceptable have been recorded in the footnotes.

(b) *English translation, notes etc.*

The English translation and explanatory and critical notes subjoined to the Sanskrit text as well as references to parallel passages given in the footnotes have been taken with necessary modi-

1. It is difficult to say whether it is *yavakoṭi* or *yamakoṭi* but most of the manuscripts give the former reading.

fications from my D. Litt. Thesis. Most of the matter in this introduction is also derived from the same source.

The question of translating technical material written in Sanskrit into English presents considerable difficulty. It requires a thorough knowledge of both the languages, which few can claim. Effort has been directed towards giving, as far as possible, a literal version of the text in English. The portions of the English translation enclosed within brackets do not occur in the text and have been given in the translation to make it understandable and are, at places, explanatory. Without these portions, the translation, at these places, might appear meaningless to a reader who cannot consult the original for lack of knowledge of Sanskrit. Attempt has been made to keep the spirit of the original and as far as possible the sequence of the text has been kept unaltered. Sanskrit technical terms having no equivalents in English have been given as such in the translation. They have been explained in the subjoined notes and the reader can always refer to the glossary of the technical terms given in the end to find the meaning of such terms whenever the subjoined notes do not contain the explanations of the terms.

Verses dealing with the same topic have been translated together and are prefixed by an introductory heading briefly summarizing their contents. This is in keeping with the practice followed by the commentators. For the convenience of the Sanskrit-knowing readers, the Sanskrit text of each passage translated has been given just before its English translation.

The translation is followed by short notes and comments comprising: (1) elucidation of the text where necessary, (2) *rationale* of the rule given in the text, (3) illustrative solved examples, where necessary, (4) critical notes, and (5) other relevant matter, depending on the passage translated. In doing so, a vast literature has been consulted and parallel passages occurring elsewhere have been noted in the footnotes. Practically all commentaries in Sanskrit on the *Āryabhaṭīya*, whether published or in manuscript, have been consulted. They have been of considerable help in translating the text; without them quite a number of passages would have remained obscure. Advantage has also been taken of the interpretations and views of the earlier translators of the *Āryabhaṭīya*, such as P.C. Sengupta and W.E. Clark.

For the convenience of the reader, the chapter-name has been mentioned at the top on the left hand page and the subject matter under discussion at the top on the right hand page. The verse-number is also mentioned at the top.

Four appendices have been given at the end :

1. Index of half-verses and key-passages.
2. Index-glossary of technical terms.
3. Subject Index.
4. Bibliography.

It is hoped that they would prove useful to the reader.

9. ACKNOWLEDGEMENT

Parts I and II of the present Series were to appear as Part One (containing a general introduction to the works of Bhāskara I) and Part Four (containing Bhāskara I's commentary and English translation of the *Āryabhaṭīya*) of the *'Bhāskara I and his works'* series, of which Parts Two and Three were published by the Department of Mathematics and Astronomy, Lucknow University, Lucknow. However, in spite of pressing demand from interested readers, these could not be published so far. I am grateful to the Indian National Science Academy, New Delhi, for sponsoring the present publication. I am greatly indebted to Professor F.C. Auluck, Vice-President, National Commission for the Compilation of the History of Sciences in India, and President, Organizing Committee for the 1500th Birth Anniversary of Āryabhaṭa I, and to Dr B.V. Subbarayappa, Executive Secretary, Indian National Science Academy, New Delhi, and Secretary, Organizing Committee for the 1500th Birth Anniversary of Āryabhaṭa I, who have taken keen interest in the present work and have gladly offered all possible help and advice from time to time. Thanks are due also to Prof. B.P. Pal, President, Indian National Science Academy, New Delhi, for writing the foreword to the present series of works.

Originally, the idea was to publish only the English translation of the *Āryabhaṭīya* along with notes and comments as was earlier done by P.C. Sengupta and W.E. Clark. The Sanskrit text was included at the suggestion of Shri K.V. Sarma, who took the responsibility of

preparing a critically collated text on the basis of the manuscripts of the *Āryabhaṭīya* that existed in the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum, and in the Library of the Government Sanskrit College, Tripunithura, and were accessible to him. The four appendices occurring at the end of this volume were also prepared by him. These have enhanced the value and usefulness of the work. Shri Sarma also gave his wholehearted cooperation in the editing of the present series of books. I offer my sincere thanks to him.

Thanks are due to the authorities of the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum, the Government Sanskrit College, Tripunithura, and the Lucknow University Library, Lucknow, whose manuscripts were utilized in the collation of the Sanskrit text, and to the many scholars whose works were consulted during the preparation of the present work.

I wish to express my deep sense of gratitude to my teacher, the late Dr. A.N. Singh, and to the late Dr. Bibhutibhusan Datta, who, in 1954, had gone through the English translation and notes and had offered valuable suggestions for their improvement. An alternative interpretation of vs. 1 of the *Gītikā-pāda*, as suggested by the latter, is mentioned in his sacred memory.

I am grateful to Dr R.P. Agarwal, Professor and Head of the Department of Mathematics and Astronomy, Lucknow University, Lucknow, also Hony. Librarian of the Lucknow University Library, Lucknow, for providing me all facilities in my work.

My cordial thanks are due to my colleague and friend Pandit Markandeya Misra, Jyotishacharya, for his assistance in the present work.

I must also express my thanks to the workers of the V.V.R.I. Press, Hoshiarpur, for the excellent composing, printing and get-up of the book.

K. S. SHUKLA

CHAPTER I

THE GĪTIKĀ SECTION

('TEN APHORISMS IN THE GĪTIKĀ STANZAS')

[In the 10 stanzas composed in the *gītikā* metre, comprising the 10 aphorisms (*sūtra*) of this chapter, Āryabhaṭa sets out the parameters which are necessary for calculations in astronomy. A beginner in astronomy was supposed to learn them by heart so that he might not feel any difficulty while making calculations later on. For the convenience of the beginner, this chapter was written as an independent tract and issued under the name *Daśagītikā-sūtra* ('Ten aphorisms in the *gītikā* stanzas') which is mentioned in the concluding stanza. When this *Daśagītikā-sūtra* is regarded as a chapter of the *Āryabhaṭīya*, it is called *Gītikā-pāda* (*Gītikā* Section).]

INVOCATION AND INTRODUCTION

प्रणिपत्यैकमनेकं कं सत्यां देवतां परं ब्रह्म ।
आर्यभटस्त्रीणि गदति गणितं कालक्रियां गोलम् ॥ १ ॥[1]

1. **Having paid obeisance to God Brahmā—who is one and many, the real God, the Supreme Brahman - Āryabhaṭa sets forth the three, *viz*., mathematics (*gaṇita*), reckoning of time (*kālakriyā*) and celestial sphere (*gola*).**

Obeisance to God Brahmā at the outset of the work points to the school to which the author Āryabhaṭa I belongs. "Obeisance has been paid to Svayambhū (Brahmā)", writes the commentator

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1. *Abbreviations* : Text mss. A to G. Text in later works and commentaries : Bh. (Bhāskara I), Br. (Brahmagupta), Go. (Govindasvāmī), Kṛ. (Kṛṣṇadāsa), Nī. (Nīlakaṇṭha), Pa. (Parameśvara), Pṛ. (Pṛthūdaka), Ra. (Raghunātha-rāja), Śa. (Śaṅkaranārāyaṇa), So. (Someśvara), Sū. (Sūryadeva), Ud. (Udayadivākara), Ya. (Yallaya).

Sūryadeva (b. A.D. 1191), "because the science which is being set out was due to Him and the mysteries of that science were revealed to Āryabhaṭa on worshipping Him."

Brahmā is spoken of as one and many, because, as writes the commentator Bhāskara I (A.D. 629), when viewed as the unchangeable (*nirvikāra*) and unstained (*nirañjana*) God, He is one, but when taken to reside in the bodies of so many living beings, He is many ; or, in the beginning He was only one, but later He became twofold—man and woman—and created all living beings and became many ; or, viewed as the omnipresent God (*viśvarūpa*), He is unquestionably one and many. He is called the 'real god' (*satyā devatā*), because the other gods having been created by Him are not real gods. He is called the 'Supreme Brahman' (*paraṁ brahma*), because He is the root cause of the world.

Bhāskara I thinks that the first half of the stanza may be interpreted also as obeisance to the two *Brahmans*—the *Śabda-Brahman* (*satyā devatā*) and the *Para-Brahman* ; or else, as obeisance to the triad, Hiraṇyagarbha (the Supreme Body), consisting of the subtle bodies of all living beings taken collectively), the Causative Power of the Supreme Body (*satyā devatā*), and the Master of that Power (*Para-Brahma*, the Supreme Brahman). For details, the reader is referred to Bhāskara I's commentary on the above stanza (in Vol. II).

According to Bibhutibhushan Datta, *kaṁ* in the text may be interpreted as *ānandakaṁ* (meaning 'supreme bliss'), *satyāṁ* as *sat-svarūpaṁ* (meaning 'really existent truth'), and *devatāṁ* as *cit-svarūpaṁ* meaning 'pure intelligence'). The text should, then, be translated as :

> "Having paid obeisance to the Supreme Brahmā who is one and also many, who is supreme bliss, really existent truth, and pure intelligence—Āryabhaṭa sets forth the three, *viz.*, mathematics (*gaṇita*), reckoning of time (*kālakriyā*), and celestial sphere (*gola*)."

METHOD OF WRITING NUMBERS

वर्गाक्षराणि वर्गेऽवर्गेऽवर्गाक्षराणि कात् ङ्मौ यः ।
खद्विनवके स्वरा नव वर्गेऽवर्गे नवान्त्यवर्गे वा ॥ २ ॥

2. **The *varga* letters (*k* to *m*) (should be written) in the *varga* places and the *avarga* letters (*y* to *h*) in the *avarga* places. (The *varga* letters take the numerical values 1, 2, 3, etc.) from *k* onwards; (the numerical value of the initial *avarga* letter) *y* is equal to *ṅ* plus *m* (*i.e.*, 5+25). In the places of the two nines of zeros (which are written to denote the notational places), the nine vowels should be written (one vowel in each pair of the *varga* and *avarga* places). In the *varga* (and *avarga*) places beyond (the places denoted by) the nine vowels too (assumed vowels or other symbols should be written, if necessary).**

In the Sanskrit alphabet the letters *k* to *m* have been classified into five *vargas* (classes)— *ka-varga*, *ca-varga*, *ṭa-varga*, *ta-varga* and *pa-varga*. These letters are therefore referred to above as *varga* letters. These are supposed to bear the numerical values 1 to 25 as shown in the following table :[1]

Table 1. *Varga* letters and their numerical values

Varga	Letters and their numerical values				
ka-varga	*k* = 1,	*kh* = 2,	*g* = 3,	*gh* = 4,	*ṅ* = 5,
ca-varga	*c* = 6,	*ch* = 7,	*j* = 8,	*jh* = 9,	*ñ* = 10,
ṭa-varga	*ṭ* = 11,	*ṭh* = 12,	*ḍ* = 13,	*ḍh* = 14,	*ṇ* = 15,
ta-varga	*t* = 16,	*th* = 17,	*d* = 18,	*dh* = 19,	*n* = 20,
pa-varga	*p* = 21,	*ph* = 22,	*b* = 23,	*bh* = 24,	*m* = 25.

The letters *y* to *h* are called *avarga* letters, because they are not classified into *vargas* (classes or groups). These letters bear the following numerical values :

y = 30, *r* = 40, *l* = 50, *v* = 60, *ś* = 70, *ṣ* = 80, *s* = 90, *h* = 100.

1. The word *kāt* in the text is meant to show that in this system the *varga* letters take the numerical values 1, 2, 3, ... beginning with *k* and not with *k*, *ṭ*, *p* and *y* as in the case of the *kaṭapayādi* system and that *ñ* and *n* are not zero in this system.

The values of the said *avarga* letters are taken to increase by 10 because the *avarga* letters are written in the *avarga* places, and increase by 1 in the *avarga* place means increase by 10 in the *varga* place.

On the analogy of the *varga* and *avarga* classification of the letters, the notational places are also divided into the *varga* and *avarga* places. The odd places denoting the units' place, the hundreds' place, the ten thousands' place and so on, are called the *varga* places (because 1, 100, 10000, etc. are perfect squares) ; and the even places denoting the tens' place, the thousands' place, and so on are called the *avarga* places (because 10, 1000, etc., are non-square numbers).

The text says that the *varga* letters should be written down in the *varga* places and the *avarga* letters in the *avarga* places. But how ? This is explained below :

The notational places are written first. The usual practice in India is to denote them by ciphers :

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Instead, it is suggested that they should be denoted by the nine vowels[1] (*a, i, u, ṛ, ḷ, e, o, ai, au*) in the following manner :

au au ai ai o o e e ḷ ḷ ṛ ṛ u u i i a a

When a letter is joined with a vowel (for example, in *gṛ* the letter *g* is joined with the vowel *ṛ*), the letter denotes a number and the vowel the place where that number is to be written down. Thus *gṛ* stands for the number *g* (=3) written in the *varga* place occupied by the vowel *ṛ* in the *varga* place as below : (A=*Avarga*, V=*Varga*)

A	V	A	V	A	V	A	V
ṛ	ṛ	u	u	i	i	a	a
	g	0	0	0	0	0	0
=	3	0	0	0	0	0	0

Thus *gṛ*=3000000. *g* has been written in the *varga* place because *g* is a *varga* letter.

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1. It is immaterial whether the short vowels *a, i, u,* etc. are used or the long vowels *ā, ī, ū,* etc. Thus, in a number-chronogram, a letter joined with a short vowel means the same thing as the same letter joined with the same long vowel. Thus *ka*=*kā*=1, *ki*=*kī*=100, and so on.

Similarly, *ṅiśibuṇḷṣkhṛ* $(=\dot{n}+i,\ \acute{s}+i,\ b+u,\ ṇ+ḷ,\ ṣ+kh+ṛ)$ denotes the number which is obtained by writing *ṅ* in the *varga* place and *ś* in the *avarga* place occupied by the vowel *i* ; *b* in the *varga* place occupied by the vowel *u* ; *ṇ* in the *avarga* place occupied by the vowel *ḷ* ; and *ṣ* in the *avarga* place and *kh* in the *varga* place occupied by the vowel *ṛ* as follows :

ḷ	ḷ	ṛ	ṛ	u	u	i	i	a	a
ṇ	*ṣ*	*kh*	0	*b*	*ś*	*ṅ*	0	0	
=1	5	8	2	2	3	7	5	0	0

Thus *ṅiśibuṇḷṣkhṛ*=1582237500.

The rule stated in the above stanza is meant essentially to provide a key to decipher the numerical values borne by the letter chronograms used by the author in the succeeding stanzas. The commentator Sūrya-deva (b. A.D. 1191), therefore, interprets the above stanza as follows :

> "The *varga* letters denoting numbers which occur in the *Gīti-sūtras* that follow should be written in the odd places, and the *avarga* letters should be written in the even places ... "

The instruction *'ṅmau yaḥ'* serves two purposes. Firstly, it gives the value of the letter *y* as equal to *ṅ* plus *m* (=5+25=30) ; secondly, it suggests that the conjoint letter *ṅm* means *ṅ*+*m*.

The statement of "two nines of zeros" in the text refers to the Indian method of writing the notational places by means of zeros. In the present primary schools in India when a student is taught to write large numbers he is first made to write the notational places by means of zeros arranged horizontally as follows :

0 0 0 0 0 0 0 0 0

The teacher then points to the first zero on the right and says "units' place", then to the next zero and says "tens' place" then to the next zero and says "hundreds' place", and so on. This practice of writing the notational places is of immemorial antiquity in India. It has been mentioned by the commentator Bhāskara I (A.D. 629), who says :

"Writing down the places, we have 0 0 0 0 0 0 0 0 0 0."

REVOLUTION-NUMBERS AND ZERO POINT

युगरविभगणाः ख्युघृ, शशि
चयगियिङुशुछ्लृ, कु ङिशिबुण्लृष्खृ[1] प्राक् ।
शनि ढुङ्विघ्व, गुरु ख्रि-
च्युभ, कुज भद्लिझ्नुखृ, भृगुबुधसौराः ॥ ३ ॥

चन्द्रोच्च [2]र्जुष्खिध, बुध
सुगुशिथृन, भृगु जषबिखुछृ, शेषार्काः ।
बुफिनच पातविलोमा,
बुधाह्न्यजार्कोदयाच्च लङ्कायाम् ॥ ४ ॥

3-4. In a *yuga*, the eastward revolutions of the Sun are 43,20,000 ; of the Moon, 5,77,53,336 ; of the Earth,[3] 1,58,22,37,500 ; of Saturn, 1,46,564 ; of Jupiter, 3,64,224 ; of Mars, 22,96,824 ; of Mercury and Venus, the same as those of the Sun ; of the Moon's apogee, 4,88,219 ; of (the *śīghrocca* of) Mercury, 1,79,37,020 ; of (the *śīghrocca* of Venus, 70,22,388 ; of (the *śīghroccas* of) the other planets, the same as those of the Sun ; of the moon's ascending node in the opposite direction (*i.e.*, westward), 2,32,226.[4] These revolutions commenced at the beginning of the sign Aries on Wednesday at sunrise at Laṅkā (when it was the commencement of the current *yuga*).

The 'Moon's apogee' is that point of the Moon's orbit which is at the remotest distance from the Earth, and the 'Moon's ascending node' is that point of the ecliptic where the Moon crosses it in its northward motion.

The *śīghroccas* of Mercury and Venus are the imaginary bodies which are supposed to revolve around the Earth with the heliocentric mean angular velocities of Mercury and Venus, respectively, their directions from the Earth being always the same as those of the mean

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1. Go. ख्षृ.
2. C.D. Kṛ. Pa. Sū. ज्रुष्खिध ; Bh. Nī. Pa. (alt.), Ra. So. र्जुष्खिध.
3. These are the rotations of the Earth, eastward.
4. These very revolutions, excepting those of the Earth, are stated in *MBh*, vii. 1-5 ; *LBh*, i. 9-14 ; and *ŚiDVṛ*, *Grahagaṇita*, i. 3-6.

positions of Mercury and Venus from the Sun. It will thus mean that the revolutions of Mars, the *śīghrocca* of Mercury, Jupiter, the *śīghrocca* of Venus, and Saturn, given above, are equal to the revolutions of Mars, Mercury, Jupiter, Venus and Saturn, respectively, round the Sun.

The following table gives the revolutions of the Sun, the Moon and the planets along with their periods of one sidereal revolution. The sidereal periods according to the Greek astronomer Ptolemy (A.D. c. 100-c. 178) and the modern astronomers are also given for the sake of comparison.

Table 2. Mean motion of the planets

Planet	Revolutions in 43,20,000 years	Sidereal period in terms of days		
		Āryabhaṭa I	Ptolemy[1]	Moderns[2]
Sun	43,20,000	365·25868	365·24666	365·25636
Moon	5,77,53,336	27·32167	27·32167	27·32166
Moon's apogee	4,88,219	3231.98708	3231·61655	3232·37543
Moon's asc. node	2,32,226	6794·74951	6796·45587	6793·39108
Mars	22,96,824	686·99974	686·94462	686·9797
Śīghrocca of Mercury	1,79,37,020	87·96988	87·96935	87·9693
Jupiter	3,64,224	4332·27217	4330·96064	4332·5887
Śīghrocca of Venus	70,22,388	224·69814	224·69890	224·7008
Saturn	1,46,564	10766·06465	10749·94640	10759·201

The epoch of the planetary motion mentioned in the text marks the beginning of the current *yuga* and not the beginning

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1. Taken from Bina Chatterjee, "*The Khaṇḍa-khādyaka of Brahmagupta*", World Press, Calcutta, 1970, vol. I, Appendix VII, p. 281.

2. Taken from H.N. Russell, Dugan and J.Q. Stewart, *Astronomy*, Part I : *The Solar system*, Revised edition, Ginn and Company, Boston, Appendix. Also, see *ibid.*, pp. 150, 159. The sidereal periods of Moon's apogee and ascending node are taken from P.C. Sengupta and N.C. Lahiri's introduction (p. xiv) to Babuāji Miśra's edition of Śrīpati's *Siddhānta-śekhara*.

of the current *Kalpa* as was supposed by P.C. Sengupta. The current *Kalpa*, according to Āryabhaṭa I, started on Thursday 1,98,28,80,000 years or 7,24,26,41,32,500 days before the beginning of the current *yuga* ; and 1,98,61,20,000 years or 7,25,44,75,70,625 days before the beginning of the current Kaliyuga.[1] The current Kaliyuga began on Friday, February 18, 3102 B.C., at sunrise at Laṅkā (a hypothetical place on the equator where the meridian of Ujjain intersects it), which synchronized with the beginning of the light half of the lunar (synodic) month of Caitra.

One thing that deserves special notice is the statement of the Earth's rotations. Āryabhaṭa I is, perhaps, the earliest astronomer in India who advanced the theory of the Earth's rotation and gave the number of rotations that the Earth performs in a period of 43,20,000 years. The period of one sidereal rotation of the Earth according of Āryabhaṭa I is $23^h\ 56^m\ 4^s{\cdot}1$. The corresponding modern value is $23^h\ 56^m\ 4^s{\cdot}091$.[2] The accuracy of Āryabhaṭa I's value is remarkable.

Of the other Indian astronomers who upheld the theory of the Earth's rotation, mention may be made of Pṛthūdaka (A.D. 860) and Makkibhaṭṭa (A.D. 1377). In the *Skanda-purāṇa* (1. 1. 31. 71), too, the Earth is described as revolving like a *bhramarikā* (spinning top, potter's wheel or whirlpool).

The commentators of the *Āryabhaṭīya*, who hold the opinion that the Earth is stationary, think that Āryabhaṭa I states the rotations of the Earth because the asterisms, which revolve westward around the earth by the force of the provector wind, see that the Earth rotates eastward.

These commentators were indeed helpless because Āryabhaṭa I's theory of the Earth's rotation received a severe blow at the hands of Varāhamihira (d. A.D. 587) and Brahmagupta (A.D. 628) whose arguments against this theory could not be refuted by any Indian astronomer.

It is noteworthy that the Greek astronomer Ptolemy, following Aristotle (B.C. 384-322), believed that the Earth was stationary and adduced arguments in support of his view.

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1. *Vide infra* notes on verse 5.

2. See W. M. Smart, *Text-Book on Spherical Astronomy*, Cambridge, 1940, p. 420.

KALPA, MANU AND BEGINNING OF KALI

काहो मनवो ढ, मनुयु-
गाः[1] श्ख, गतास्ते च, मनुयुगाः[2] छ्ना च ।
कल्पादेर्युगपादा
ग च, गुरुदिवसाच्च, भारतात् पूर्वम् ॥ ५ ॥

5. **A day of Brahmā (or a Kalpa) is equal to (a period of) 14 Manus, and (the period of one) Manu is equal to 72 *yugas*. Since Thursday, the beginning of the current Kalpa, 6 Manus, 27 *yugas* and 3 quarter *yugas* had elapsed before the beginning of the current Kaliyuga (lit. before Bhārata).**

Thus we have

1 Kalpa = 14 Manus

and 1 Manu = 72 *yugas*,

so that 1 Kalpa = 1008 *yugas* or 4,35,45,60,000 years.

Likewise, the time elapsed since the beginning of the current Kalpa up to the beginning of the current Kaliyuga

$= 6$ Manus $+ 27\frac{3}{4}$ *yugas*

$= (6 \times 72 + 27\frac{3}{4})$ *yugas*

$= (432 + 27\frac{3}{4}) \times 4320000$ years

$= 1986120000$ years or 725447570625 days.

It is interesting to note that Āryabhaṭa I prefers to say "before Bhārata" (*bhāratāt pūrvam*) instead of saying "before the beginning of Kaliyuga" which is the sense actually intended here.

Regarding the interpretation of *bhāratāt pūrvam* there is difference of opinion amongst the commentators. The commentator Someśvara interprets it as meaning 'before the occurrence of the Bhārata (battle)'. P. C. Sengupta (A.D. 1927) and W.E. Clark (A.D. 1930), too,

1. Bh. Śa. मनुयुगाः ; all others मनुयुग.
2. E. Bh. Śa. युगाः ; all others युग.

have interpreted the word *bhārata* as meaning 'the Bhārata battle'. In the *Mahābhārata* we are told that the Bhārata battle occurred at the end of the *Dvāpara yuga* and before the beginning of the *Kali yuga* :

> "The battle between the armies of the Kurus and the Pāṇḍavas occurred at Syamantapañcaka (Kurukṣetra) when it was the junction of Kali and Dvāpara."[1]

So this interpretation of *bhāratāt pūrvam* ('before Bhārata') is equivalent to *kaliyugāt pūrvam* ('before Kaliyuga'), as it ought to be.

The commentators Bhāskara I (A.D. 629), Sūryadeva (b. A.D. 1191) and others have interpreted *bhāratāt pūrvam* as meaning 'before Yudhiṣṭhira', *i.e.*, 'before the time when Yudhiṣṭhira of the Bharata dynasty relinquished kingship and proceeded on the last journey (*mahā-prasthāna*)'.[2] According to these commentators, this event

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1. *Mahābhārata*, *Ādiparva*, ch. 2, vs. 13.

2. According to the *Bhāgavata-Purāṇa* (*Skandha* 1, ch. 15, vs. 36), Kaliyuga began on the day on which Lord Kṛṣṇa left this earthly abode :

यदा मुकुन्दो भगवानिमां महीं जहौ स्वतन्वा श्रवणीयसत्कथः ।
तदाहरेवाप्रतिबुद्धचेतसामधर्महेतुः कलिरन्ववर्तत ॥

And when Yudhiṣṭhira came to know that Kaliyuga had commenced, he made up his mind to proceed on the last journey (*Skandha* 1, ch. 15, vs. 37) :

युधिष्ठिरस्तत्परिसर्पणं बुधः पुरे च राष्ट्रे च गृहे तथाऽऽत्मनि ।
विभाव्य लोभानृतजिह्महिंसनाद्यधर्मचक्रं गमनाय पर्यधात् ॥

Other views are that Kaliyuga commenced the moment Lord Kṛṣṇa left for heaven :

यस्मिन् कृष्णो दिवं यातस्तस्मिन्नेव तदाहनि ।
प्रतिपन्नं कलियुगमिति प्राहुः पुराविदः ॥

Bhāgavata-Purāṇa, *Skandha* 12, ch. 2, vs. 33

Or, when the Seven Ṛṣis (*i.e.*, the seven stars of the constellation of Ursa Major) entered the asterism Maghā :

यदा देवर्षयः सप्त मघासु विचरन्ति हि ।
तदा प्रवृत्तस्तु कलिर्द्वादशाब्दशतात्मकः ॥

***Bhāgavata-Purāṇa*, *Skandha* 12, ch. 2, vs. 31**

took place on Thursday, the last day of the past Dvāpara. But the basis of this assumption is not specified. The commentators simply say : "This is what is well known." (*iti prasiddhiḥ*).[1]

According to these commentators, too, ***bhāratāt pūrvam*** ultimately means 'before the beginning of the current Kaliyuga'.

Brahmagupta criticises Āryabhaṭa I for his teaching in the above stanza. Writes he :

> "Since the measures of a Manu, a (quarter) *yuga* and a Kalpa and the periods of time elapsed since the beginnings of Kalpa and Kṛtayuga (as taught by Āryabhaṭa) are not in conformity with those taught in the Smṛtis, it follows that Āryabhaṭa is not aware of the mean motions (of the planets)."[2]
>
> "Since Āryabhaṭa states that three quarter *yugas* had elapsed at the beginning of Kaliyuga, the beginning of the current *yuga* and the end of the past *yuga* (according to him) occurred in the midst of Kṛtayuga; so his *yuga* is not the true one."[3]
>
> "Since the initial day on which the Kalpa started according to (Āryabhaṭa's) sunrise system of astronomy is Thursday and not Sunday (as it ought to be), the very basis has become discordant."[4]

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1. It is, however, noteworthy that Indian astronomers of all schools are perfectly unanimous in taking Friday as the day on which the current Kaliyuga commenced.

2. न समा मनुयुगकल्पाः कल्पादिगतं कृताद्ियातं च ।
स्मृत्युक्तैरार्यभटो नातो जानाति मध्यगतिम् ॥

BrSpSi, xi. 10

3. आर्यभटो युगपादांस्त्रीन् यातानाह कलियुगादौ यत् ।
तस्य कृतान्तर्यस्मात् स्वयुगाद्यन्तौ न सत् तस्मात् ॥

BrSpSi, xi. 4

4. ऊँकारो दिनवारो गुरुरौदयिकोऽस्य भवति कल्पादौ ।
न भवत्यर्को यस्मादोङ्कारे विस्वरस्तस्मात् ॥

BrSpSi, xi. 11

In reply to this criticism, astronomer Vaṭeśvara (A. D. 904) says :

"If the *yuga* stated by Brahmagupta conforms to the teachings of the Smṛtis, how is it that the Moon (according to him) is not beyond the Sun (as stated in the Smṛtis). If that is unacceptable because that statement of the Smṛtis is false, then, alas, the *yuga*-hypothesis of the Smṛtis, too, is false."[1]

"Since a planet does not make complete revolutions during the quarter *yugas* acceptable to Brahmagupta, son of Jiṣṇu, (whereas it does during the quarter *yugas* according to Āryabhaṭa), it follows that the quarter *yugas* of Śrīmad Āryabhaṭa (and not those of Brahmagupta) are the correct ones."[2]

"If a Kalpa should begin with a Sunday, how is it that Brahmagupta's Kalpa does not end with a Saturday. Brahmagupta's Kalpa being thus contradictory to his own statement, it is a fabrication of his own mind (and is by no means authoritative)."[3]

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1. स्मार्तमस्य युगमेव चेत्कथं नो रवेरुपरि शीतदीधितिः ।
   तत्स्मृतावसदितीह नेष्यते हन्त ! साऽपि युगकल्पना मुधा ॥

   VaSi, Grahagaṇita, ch. 1, sec. 10, vs. 3

2. जिष्णुपुत्रकथितैर्युगाङ्घ्रिभिः खेचरो नहि यतः स्वपर्ययम् ।
   भुञ्जते सममतो युगाङ्घ्रयः श्रीमदार्यभटकीर्तिताः स्फुटाः ॥

   VaSi, Grahagaṇita, ch. 1, sec. 10, vs. 2

3. कल्पादौ यद्यर्कः कल्पान्ते भास्करिः कथं न भवेत् ।
   निजवचनव्याघातात्स्वबुद्धिकल्पः कृतः कल्पः ॥

   VaSi, Grahagaṇita, ch. I, sec. 10, vs. 10

PLANETARY ORBITS, EARTH'S ROTATION

शशिराशयष्ठ चक्रं
तेंऽशकलायोजनानि य-व-ञ-गुणाः ।
प्राणेनैति कलां भूः,[1]
खयुगांशे ग्रहजवो, भवांशोऽर्कः ॥ ६ ॥

6. **Reduce the Moon's revolutions (in a *yuga*) to signs, multiplying them by 12 (lit. using the fact that there are 12 signs in a circle or revolution). Those signs mutiplied successively by 30, 60 and 10 yield degrees, minutes and *yojanas*, respectively. (These *yojanas* give the length of the circumference of the sky). The Earth rotates through (an angle of) one minute of arc in one respiration (=4 sidereal seconds). The circumference of the sky divided by the revolutions of a planet in a *yuga* gives (the length of) the orbit on which the planet moves.[2] The orbit of the asterisms divided by 60 gives the orbit of the Sun.[3]**

Thus we have

Orbit of the sky $= 57753336 \times 12 \times 30 \times 60 \times 10$ *yojanas*
$= 12474720576000$ *yojanas*

Orbit of the asterisms $= 173260008$ *yojanas*

Orbit of the Sun $= 2887666\frac{4}{5}$ *yojanas*

Orbit of the Moon $= 216000$ *yojanas*

Orbit of Mars $= 5431291\frac{132027}{287103}$ *yojanas*

Orbit of (*Śīghrocca of*) Mercury $= 695473\frac{373277}{896851}$ *yojanas*

Orbit of Jupiter $= 34250133\frac{699}{1897}$ *yojanas*

Orbit of (*Śīghrocca* of) Venus $= 1776421\frac{255221}{585199}$ *yojanas*

Orbit of Saturn $= 85114493\ \frac{5987}{36641}$ *yojanas*.

---

1. Br. Pṛ. Ud. भूः ; all others भं.
2. *Cf.* Someśvara : ग्रहजवो ग्रहपरिधिः ग्रहकक्ष्येत्यर्थः ।
3. The same rule, excepting the rate of the Earth's motion, occurs in *MBh*, vii. 20, also.

These orbits are hypothetical and are based on the following two assumptions :

1. That all the planets have equal linear motion in their respective orbits.[1]

2. That one minute of arc (1′) of the Moon's orbit is equal to 10 *yojanas* in length.[2]

From the second assumption, the length of the Moon's orbit comes out to be 216000 *yojanas*. Multiplying this by the Moon's revolution-number (*viz.* 57753336), we get 12474720576000 *yojanas*. This is the distance described by the Moon in a *yuga*. From the first assumption, this is also the distance described by any other planet in a *yuga*. Hence

$$\text{Orbit of a planet} = \frac{\text{distance described by a planet in a } yuga}{\text{revolution-number of that planet}}.$$

This is how the lengths of the orbits of the various planets stated above have been obtained.

In the case of the asterisms, it is assumed that their orbit is 60 times the orbit of the Sun. By saying that "the orbit of the asterisms divided by 60 gives the orbit of the Sun", Āryabhaṭa I really means to say that "the orbit of the asterisms is 60 times the orbit of the Sun."

Indian astronomers, particularly the followers of Āryabhaṭa I, believe that the distance described by a planet in a *yuga* denotes the circumference of the space, supposed to be spherical, which is illumined by the Sun's rays. This space, they call 'the sky' and its circumference 'the orbit of the sky'. Bhāskara I says :

> "(The outer boundary of) that much of the sky as the Sun's rays illumine on all sides is called the clrcumference or orbit of the sky. Otherwise, the sky is beyond limit; it is impossible to state its measure."[3]
>
> "For us the sky extends to as far as it is illumined by the rays of the Sun. Beyond that, the sky is immeasurable."[4]

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1. See *Ā*, iii, 12.
2. This is implied in the text under discussion.
3. See Bhāskara I's commentary on *Ā*, i. 6, in Vol. II.
4. See Bhāskara I's commentary on *Ā*, iii. 12, in Vol. II.

According to the Indian astronomers, therefore,

$$\text{Orbit of a planet} = \frac{\text{Orbit of the sky}}{\text{Planet's revolution-number}}$$

The statement of the Earth's rotation through 1′ in one respiration,[1] stated in the text, has been criticised by Brahmagupta, who says :

> "If the Earth moves (revolves) through one minute of arc in one respiration, from where does it start its motion and where does it go ? And, if it rotates (at the same place), why do tall lofty objects not fall down ?"[2]

The reading *bhaṁ* (in place of *bhūḥ*) adopted by the commentators is evidently incorrect. The correct reading is *bhūḥ,* which has been mentioned by Brahmagupta (A.D. 628), Pṛthūdaka (A. D. 860) and Udayadivākara (A.D. 1073).[3]

LINEAR DIAMEIERS

नृ-षि योजनं, ञिला भू-
व्यासो, ऽर्केन्द्वोर्घ्रिञा गिण, क मेरोः ।
भृगु-गुरु-बुध-शनि-भौमाः
शशि-ङ-ञ-ण-न-मांशकाः, समार्कसमाः ॥ ७ ॥

7. **8000** *nṛ* **make a** *yojana*. **The diameter of the Earth is 1050** *yojanas*; **of the Sun and the Moon, 4410 and 315** *yojanas*, **(respectively) ;[4] of Meru, 1** *yojana* **; of Venus, Jupiter, Mercury, Saturn and Mars (at the Moon's mean distance), one-fifth, one-tenth, one-fifteenth, one-twentieth, and one-twentyfifth, (respectively), of the Moon's diameter.[5] The years (used in this work) are solar years.**

---

1. 1 respiration = 4 seconds of time.
2. प्राणेनैति कलां भूर्यदि तर्हि कुतो व्रजेत् कमध्वानम् ।
   आवर्तनमुर्व्याश्चेन्न पतन्ति समुच्छ्रयाः कस्मात् ॥
   BrSpSi, xi. 17.
3. See his commentary on *LBh,* i. 32-33.
4. The same values are given in *MBh,* v. 4 ; *LBh,* iv. 4.
5. *Cf. MBh,* vi. 56.

Nṛ is a unit of length whose measure is equal to the height of a man. *Nṛ* is also known as *nara*, *puruṣa*, *dhanu* and *daṇḍa*, "*Puruṣa*, *dhanu*, *daṇḍa* and *nara* are synonyms", says Bhāskara I.

The diameters of the Earth, the Sun, the Moon, and the Planets stated above may be exhibited in the tabular form as follows :

Table 3. Linear diameters of the Earth etc.

	Linear diameter in *yojanas*	Linear diameter in *yojanas* (at the Moon's mean distance)
Earth	1050	
Sun	4410	
Moon	315	
Mars		12.60
Mercury		21.00
Jupiter		31.50
Venus		63.00
Saturn		15.75

The following is a comparative table of the mean angular diameters of the planets :

Table 4. Mean angular diameters of the Planets

Planet	Mean angular diameter according to		
	Āryabhaṭa I	Greek astronomers[1]	Modern
Moon	31′ 30″	35′ 20″ (Ptolemy) Tycho Brahe (1546-1631)	31′ 8″
Mars	1′ 15″·6	1′ 40″	
Mercury	2′ 6″	2′ 10″	
Jupiter	3′ 9″	2′ 45″	
Venus	6′ 18″	3′ 15″	
Saturn	1′ 34″·5	1′ 50″	

1. See E. Burgess, *Translation of the Sūrya-siddhānta*, Reprint, Calcutta, 1935, p. 196 ; and introduction by P.C. Sengupta, p. xlvi.

P. C. Sengupta translates the second half of the stanza as follows :

> "The diameters of Venus, Jupiter, Mercury, Saturn and Mars are, respectively, 1/5, 1/10, 1/15, 1/20 and 1/25 of the diameter of the Moon, when taken at the mean distance of the Sun."

This is incorrect, because :

1. "When taken at the mean distance of the Sun" is not the correct translation of *samārkasamāḥ*. The correct translation is : "The years are solar years" as interpreted by Bhāskara I and Someśvara ; or "The years of a *yuga* are equal to the number of revolutions of the Sun in a *yuga*"[1] as translated by Clark and as interpreted by Sūryadeva, Parameśvara and Raghunātha-rāja.

2. The diameters of the planets stated in the stanza under consideration correspond to the mean distance of the Moon and not to the mean distance of the Sun as Sengupta has supposed. Sengupta's disagreement on this point from the commentator Parameśvara is unwarranted. All commentators agree with Parameśvara.

OBLIQUITY OF ECLIPTIC AND INCLINATIONS OF ORBITS

भापक्रमो ग्रहांशाः,
शशिविक्षेपोऽपमण्डलात् भार्धम् ।
शनि-गुरु-कुज ख-क-गार्धं,
भृगु-बुध ख, स्वाङ्गुलो घहस्तो ना ॥ ८ ॥

8. **The greatest declination of the Sun is 24°.[2] The greatest celestial latitude (lit. deviation from the ecliptic) of the Moon is $4\frac{1}{2}$°;[3] of Saturn, Jupiter and Mars, 2°, 1° and $1\frac{1}{2}$° respectively;**

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1. समार्कसमाः, समा वर्षं युगसम्बन्धि अर्कसमा अर्कभगणसमाः ।

2. The same value is given in *MBh*, iii. 6 ; *LBh*, ii. 16 ; *KK*, Part 1, iii. 7 ; *KR*, i. 50.

3. The same value occurs in *MBh*, v. 30 ; *LBh*, iv. 8 ; *KK*, Part 1, iv. 1 (*c-d*) ; *KR*, ii. 3 (*a-b*).

and of Mercury and Venus (each), 2°.[1] **96** ***aṅgulas*** **or 4 cubits make a** ***nṛ*****.**

The greatest declination of the Sun is the obliquity of the ecliptic. According to Āryabhaṭa I and other Indian astronomers, its value is 24°.[2] According to the modern astronomers its value is 23° 27′ 8″.26—46″.84 T, where T is measured in Julian centuries from 1900 A.D. The value in common use is 23½°.

The greatest celestial latitude of a planet is the inclination of the planet's orbit to the ecliptic. The values of the inclinations of the orbits of the Moon and the planets as given in the above stanza and those given by the Greek astronomer Ptolemy and the modern astronomers are being exhibited in the following table :

Table 5. Inclinations of the Orbits

Planet	Inclination of the orbit			
	Āryabhaṭa I	Ptolemy[3]	*PauSi*[4] and *RoSi*[5] of Varāhamihira	Modern[6]
Moon	4° 30′	5°	4° 40′	5° 9′
Mars	1° 30′	1°		1° 51′ 01″
Mercury	2°	7°		7° 00′ 12″
Jupiter	1°	1° 30′		1° 18′ 28″
Venus	2°	3° 30′		3° 23′ 38″
Saturn	2°	2° 30′		2° 29′ 20″

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1. The same values occur in *MBh*, vii. 9 ; *LBh*, vii. 7 (*a-b*) ; *KK*, Part 1, viii. 1 (*c-d*) ; *KR*, vii. 8 (*c-d*).

2. According to the Greek astronomer Ptolemy, the obliquity of the ecliptic is 23° 51′ 20″. See *Great Books of the Western World*, vol. 16 : *The Almagest of Ptolemy*, translated by R. Catesby Taliaferro, Book II, p. 31.

3. See E. Burgess, *Translation of the Sūrya-siddhānta*, Reprint, Calcutta, 1935, p. 52. In the *Almagest* of Ptolemy, translated R. Catesby Taliaferro, the obliquities of the epicycles of Mercury and Venus are stated as 6° 15′ and 2° 30′ respectively. See pp. 435 and 433.

4. See *PSi*, iii. 31.

5. See *PSi*, viii. 11.

6. See H.N. Russell, R.S. Dugan and J.Q. Stewart, *Astronomy*, Part I, *The Solar system*, Revised edition, Ginn and Company, Boston, Appendix.

In the case of Mercury and Venus, Āryabhaṭa I's values differ significantly from those of Ptolemy and modern astronomers because the values given by Āryabhaṭa I are geocentric and those given by Ptolemy and modern astronomers are heliocentric.

Combining the instruction in the last quarter of the above verse with that in the first quarter of verse 7, we have

24 *aṅgulas* = 1 cubit (*hasta*)
4 cubits = 1 *nṛ*
8000 *nṛ* = 1 *yojana*.

Since earth's (equatorial) diameter = 1050 *yojanas* (vs. 7), according to Āryabhaṭa I, and = 12757 km or 7927 miles, according to modern astronomy, it follows that Āryabhaṭa I's *yojana* is approximately equal to $12\frac{1}{9}$ km or $7\frac{1}{2}$ miles. Likewise his *nṛ* = 152 cm. or 5 ft approx, and cubit = $1\frac{1}{4}$ ft. approx. The length of a cubit in common use is $1\frac{1}{2}$ ft.

ASCENDING NODES AND APOGEES (APHELIA)

बुध-भृगु-कुज-गुरु-शनि न-व-
रा-ष-ह[1] गत्वांशकान् प्रथमपाताः ।
सवितुरमीषां च तथा
द्वा-ञखि-सा-ह्दा[2]-ह्लच खिच्य मन्दोच्चम् ॥ ९ ॥

9. **The ascending nodes of Mercury, Venus, Mars, Jupiter and Saturn having moved to 20°, 60°, 40°, 80° and 100° respectively (from the beginning of the sign Aries) (occupy those positions);[3] and the apogees of the Sun and the same planets (*viz*., Mercury, Venus, Mars, Jupiter and Saturn) having moved to 78°, 210°, 90°, 118°, 180° and 236° respectively (from the beginning of the sign Aries) (occupy those positions).[4]**

The following table gives the longitudes of the ascending nodes and the apogees of the planets for A. D. 499 as given by Āryabhaṭa I and as calculated by modern methods. The corresponding longitudes for A.D. 150, as stated by Ptolemy are also given for comparison.

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1. A.-G. Su. नवरषह्रा ; Bh. नवराषह् ; Pṛ. नवरषाह्
2. Kṛ. ह्द.
3. The same values occur in *MBh*, vii. 10 ; *LBh*, vii. 6 (*c-d*) ; *KK*, Part 1, viii. 1 ; *KR*, vii. 8 (*a-b*).
4. The same values occur in *MBh*, vii. 11-12 (*a-b*) ; *LBh*, i. 22 (*a-b*), 18 ; *KR*, i. 10 (*c-d*) ; vii. 9 (*c-d*).

Table 6. Longitudes of the Ascending Nodes for A.D. 499

Planet	Longitudes of the ascending nodes		
	Āryabhaṭa I	Ptolemy[1] (for A.D. 150)	By modern calculation[2]
Mars	40°	25° 30′	37° 49′
Mercury	20°	10° 00′	30° 35′
Jupiter	80°	51° 00′	85° 13′
Venus	60°	55° 00′	63° 16′
Saturn	100°	183° 00′	100° 32′

Table 7. Longitudes of the Apogees for A.D. 499

Planet	Longitudes of the apogees (aphelia)			
	Āryabhaṭa I	Ptolemy[3] (for A.D. 150)	*RoSi*[4] of Varāhmihira	By modern calculation[5]
Sun	78°	65° 30′	75°	77° 15′
Mars	118°	115° 30′		128° 28′
Mercury	210°	190° 00′		234° 11′
Jupiter	180°	161° 00′		170° 22′
Venus	90°	55° 00′		290° 4′
Saturn	236°	233° 00′		243° 40′

The word *gatvā* (meaning 'having moved' or 'having moved to') is used in the text to show that the ascending nodes and the apogees

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1. See E. Burgess, *ibid.* Appendix, p. 331. According to P.C. Sengupta and N.C. Lahiri, the longitudes of the ascending nodes of Mars, Jupiter and Saturn as given by Ptolemy are 30°, 70° and 90°, respectively. See their introduction (p. xiv) to the *Siddhanta-śekhara* of Śrīpati, Part II, edited by Babuāji Miśra, Calcutta, 1947.

2. See E. Burgess, *ibid.*, Introduction by P.C. Sengupta, p. xlviii.

3. See E. Burgess, *ibid.*, Appendix, p. 331 ; and Bina Chatterjee, *The Khaṇḍakhādyaka of Brahmagupta*, with the commentary of *Bhaṭṭotpala*, vol. I, p. 283.

of the planets are not stationary but have a motion. The commentator Bhāskara I says that by teaching their motion, Āryabhaṭa I has specified by implication their revolution-numbers in a *yuga*. The aged, who preserve the tradition, says he, remember those revolution-numbers by the continuity of tradition. The period of 35750224800 years, according to the tradition, is the common period of motion (*yuga*) of the ascending nodes of all the planets, in which the ascending nodes of Mars, Mercury, Jupiter, Venus and Saturn make 2, 1, 4, 3 and 5 revolutions, respectively.

In the case of the apogees of the planets, the periods and the corresponding revolutions, as handed down to Bhāskara I by tradition, are shown in the following table :

Table 8. Periods and Revolution-numbers of the Apogees

Apogee of	Period in years	Revolution-number
Sun	119167416000	13
Mars	357502248000	59
Mercury	23833483200	7
Jupiter	3972247200	1
Venus	7944494400	1
Saturn	178751124000	59

The commentators Sūryadeva and Raghunātha-rāja have also cited the above-mentioned periods and revolution-numbers to preserve

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(*Footnotes of the last Page* :)

According to P.C. Sengupta, the longitudes of the apogees of Sun, Mars, Mercury, Jupiter, Venus and Saturn as given by Ptolemy are 65° 30′, 106° 40′, 181° 10′, 152° 9′, 46° 10′ and 224° 10′, respectively. See his introduction to E. Burgess' *Translation of the Surya-siddhānta*, pp. xlvi and xlvii. The same values are given also in *Great Books of the Western World*, vol. 16 : *The Almagest by Ptolemy* (translated by R. Catesby Taliaferro), Book XI, pp. 386-390.

4. See *PSi*, viii. 2.

5. See E. Burgess, *ibid.*, introduction by P.C. Sengupta, p. x and xlvii.

the continuity of tradition of the school. It is remarkable that the first line of the stanza :

खाकाशाष्टकृतद्विद्विव्योमेष्वद्रीषुवह्नय: ।
युगं बुधादिपातानां विद्वद्भिः परिपठ्यते ॥
एकद्वित्रिचतुष्पञ्च भगणाः परिकीर्तिताः ।
सौम्यारशुक्रजीवार्किपातानां क्रमशो युगे ॥,

which has been quoted in full by Sūryadeva, has been cited by Bhāskara I too. This means that the passage was derived from some earlier source, and the tradition mentioned in the stanza is definitely older than Bhāskara I.

Whosoever might be the founder of the tradition, it is based on the misunderstanding that the ascending nodes and the apogees, after having started their motion from the first point of Aries at the beginning of the current Kalpa, moved exactly through the degrees mentioned by Āryabhaṭa I up to 499 A.D., the epoch mentioned by Āryabhaṭa I.

The motions of the nodes and the apogees of the planets ascribed to tradition by Bhāskara I and the other commentators are much less than their actual motions. For example, the node of Mercury, which is the slowest, actually requires about 166000 years to complete a revolution.[1] Similar is the case with the apogees.

MANDA AND *ŚIGHRA* EPICYCLES
(Odd quadrants)

झार्धानि मन्दवृत्तं
शशिनश्छ, ग-छ-घ-ढ-छ-झ यथोक्तेभ्यः ।
झा[2]-ग्ड-ग्ला[3]-धं[4]-द्रुड तथा
शनि-गुरु-कुज-भृगु-बुधोच्चशीघ्रेभ्यः ॥ १० ॥

10 The *manda* epicycles of the Moon, the Sun, Mercury, Venus, Mars, Jupiter and Saturn (in the first and third anomalistic quadrants)

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1. See C.A. Young, *A Text-book on General Astronomy*, Revised edition, 1904, p. 337.

2. Bh. Pa. So. झा ; others झ.

3. Bh. G. ग्ला ; all others ग्ल.

4. Bh. and Go. धं ; all others ड्ल.

are, respectively, 7, 3, 7, 4, 14, 7 and 9 (degrees) each multiplied by 4½ (*i.e.*, 31.5, 13.5, 31.5, 18, 63, 31.5 and 40.5 degrees, respectively) ; the *śīghra* epicycles of Saturn, Jupiter, Mars, Venus and Mercury (in the first and third anomalistic quadrants) are, respectively, 9, 16, 53, 59 and 31 (degrees) each multiplied by 4½ (*i.e.*, 40.5, 72, 238.5, 265.5 and 139.5 degrees, respectively).

(Even quadrants)

मन्दात् ङ-ख-द्-ज-डा
वक्रिणां द्वितीये पदे चतुर्थे च ।
जा-णा-क्ल-छ्ल-झ्नोच्चा-
च्छीघ्रात्, गियिङश कुवायुकक्ष्यान्त्या ॥ ११ ॥

11. The *manda* epicycles of the retrograding planets (*viz.*, Mercury, Venus, Mars, Jupiter and Saturn) in the second and fourth anomalistic quadrants are, respectively, 5, 2, 18, 8 and 13 (degrees) each multiplied by 4½ (*i.e.*, 22.5, 9, 81, 36 and 58.5 degrees, respectively) ; and the *śighra* epicycles of Saturn, Jupiter, Mars, Venus, and Mercury (in the second and fourth anomalistic quadrants) are, respectively, 8, 15, 51, 57 and 29 (degrees) each multiplied by 4½ (*i.e.*, 36, 67.5, 229.5, 256.5 and 130.5 degrees, respectively).[1] 3375 is the outermost circumference of the terrestrial wind.[2]

The dimensions of the *manda* and *śīghra* epicycles are stated in terms of degrees, where a degree stands for the 360th part of the circumference of the deferent (*kakṣyāvṛtta*). Thus, when an epicycle is stated to be A°, it means that its periphery is A/360 of the circumference of the deferent.

The following table gives the *manda* and *śīghra* epicycles as stated above by Āryabhaṭa I and also those given by Ptolemy :

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1. The same values occur in *MBh*, vii. 13-16 ; *LBh*, i. 19-22 ; *KR*, viii. 10-11.

2. The same value occurs in *ŚiDVṛ*, *Golādhyāya*, v. 2.

Table 9. *Manda* and *Śīghra* epicycles of the planets

Planet	*Manda* epicycles Āryabhaṭa I Odd quadrant	Even quadrant	Ptolemy[1]	*Śīghra* epicycles Āryabhaṭa I Odd quadrant	Even quadrant	Ptolemy[2]
Sun	13°.50	13°.50	15°.00			
Moon	31°.50	31°.50	31°.40			
Mars	63°.00	81°.00	72°.00	238°.50	229°.50	237°
Mercury	31°.50	22°.50	18°.00	139°.50	130°.50	135°
Jupiter	31°.50	36°.00	33°.00	72°.00	67°.50	69°
Venus	18°.00	9°.00	15°.00	265°.50	256°.50	259°
Saturn	40°.50	58°.50	41°.00	40°.50	36°.00	39°

It is noteworthy that in stating the dimensions of the *manda* epicycles the planets have been mentioned in the order of decreasing velocities (*manda-gati-krama*), whereas in stating the dimensions of the *śīghra* epicycles they have been mentioned in the order of increasing velocities (*śīghra-gati-krama*). It is perhaps done deliberately to emphasise this point to the reader. The use of ablative in *yathoktebhyaḥ* is meant to indicate that in finding the *manda* anomaly the longitude of the apogee is to be subtracted from the longitude of the planet. Similarly, the use of the inverted forms *uccaśīghrebhyaḥ* and *uccācchīghrāt* in place of *śīghroccebhyaḥ* and *śīghroccāt*, respectively, shows, as remarked by Bhāskara I and Someśvara, that, in finding the *śīghra* anomaly, the longitude of the planet has to be subtracted from the longitude of the *śīghrocca*.

It may be pointed out that, according to Bhāskara I and Lalla, the *manda* and *śīghra* epicycles stated above correspond to the beginnings of the respective anomalistic quadrants as is evident from the rules stated in *MBh*, iv. 38-39 (*a-b*), *LBh*, ii. 31-32 and *SiDVṛ*, I, iii. 2 and explained in Bhāskara I's commentary on

1. See Bina Chatterjee, *op.cit.*, p. 284. In the *Almagest* of Ptolemy, translated by R. Catesby Taliaferro, however, the Moon's epicyclic radius is stated as $51^p\ 51'$ which yields 31°.50 as the value of the Moon's epicycle. See p. 151.

2. See Bina Chatterjee, *op. cit.*, p. 285.

Ā, iii. 12.[1] The Kerala astronomer Śaṅkaranārāyaṇa (A. D. 869) refers to some astronomers (without naming them) who said that there was also the view that the epicycles given by Āryabhaṭa I corresponded to the end-points of the anomalistic quadrants.[2] The Kerala astronomer Govinda-svāmī, who also refers to this controversy, is of the opinion that *śīghra* epicycles stated above correspond to the beginnings of the respective anomalistic quadrants, but the *manda* epicycles stated above correspond to the last points of the respective anomalistic quadrants,[3] and, consequently, he has replaced the rules referred to above by another rule which has been quoted by Udayadivākara (1073 A.D.) in his commentary on *LBh*, ii. 31-32. This controversy is due to the fact that Āryabhaṭa I himself does not specify whether the epicycles given by him correspond to the initial points or last points of the anomalistic quadrants.

Since the epicycles stated in the text correspond to the beginnings of the odd and even anomalistic quadrants, their values at other positions of the planets are to be derived by the rule of three. Bhāskara I has prescribed the following rule .[4]

Let α and β be the epicycles (*manda* or *śīghra*) of a planet for the beginnings of the odd and even anomalistic quadrants, respectively.

---

1. The commentator Sūryadeva, too, is of this view. See his comm. on *Ā*, iii. 24, p. 114.

2. See his commentary on *LBh*, ii. 32-33, where he writes :

अन्तपरिधिपक्षोऽपि विद्यत इति केचिद्वदन्ति ।

3. Govinda-svāmī has been led to this conclusion by the fact that in stating the dimensions of the *manda* epicycles Āryabhaṭa I has mentioned the planets in the order of decreasing velocities whereas in stating the dimensions of the *śīghra* epicycles he has mentioned the planets in the order of increasing velocities. See his comm. on *MBh*. iv. 38-39 (*a-b*).

4. See *MBh*, iv. 38-39 (*a-b*); *LBh*, ii. 31-32. An equivalent rule is given in *ŚiDVṛ*, *Grahagaṇita*, iii. 2.

Then

(i) If the planet be in the first anomalistic quadrant, say at P, and its anomaly be θ,

$$\text{epicycle at } P = \alpha + \frac{(\beta - \alpha) \text{ Rsin } \theta}{R}, \text{ when } \alpha < \beta$$

$$= \alpha - \frac{(\alpha - \beta) \text{ Rsin } \theta}{R}, \text{ when } \alpha > \beta$$

and (ii) If the planet be in the second anomalistic quadrant, say at Q, and its anomaly be $90° + \phi$,

$$\text{epicycle at } Q = \beta - \frac{(\beta - \alpha) \text{ Rvers } \phi}{R}, \text{ when } \alpha < \beta$$

$$= \beta + \frac{(\alpha - \beta) \text{ Rvers } \phi}{R}, \text{ when } \alpha > \beta.$$

Similarly in the third and fourth quadrants. The epicycles thus derived are called true epicycles (*spaṣṭa-* or *sphuṭa-paridhi*).

But the tabulated *manda* epicycles or the true *manda* epicycles derived from them are not the actual epicycles on which the true planet in the case of the Sun and Moon or the true mean planet in the case of the other planets is supposed to move. It is believed that they are the mean epicycles corresponding to the mean distances of the planets. In order to obtain the actual epicycles, one should either apply the formula :

$$\text{actual } manda \text{ epicycle} = \frac{\text{tabulated}^1 \text{ or true } manda \text{ epicycle}^2 \times H}{R},$$

where H is the planet's true distance in minutes obtained by the process of iteration (*asakṛtkalākarṇa* or *mandakarṇa*),[3] or apply the process of iteration.[4] In the case of the *śīghra* epicycles, however, the actual epicycles are the same as the tabulated epicycles.

1. In the case of the Sun and the Moon.
2. In the case of the other planets, Mars etc.
3. See *MBh*, iv. 9-12 ; *LBh*, ii. 6-7,
4. See *ŚiDVṛ*, *Grahagaṇita*, iii. 17.

Brahmagupta criticises Āryabhaṭa I for stating different epicycles for odd and even anomalistic quadrants. Writes he :

> "Since in (Āryabhaṭa's) sunrise system of astronomy, the epicycle which is the multipler of the Rsine of anomaly in the odd anomalistic quadrant is different from the epicycle which is the multiplier of the Rsine of anomaly in the even anomalistic quadrant, the (*manda* or *śīghra*) correction for the end of an odd anomalistic quadrant is not equal to that for the beginning of the (next) even anomalistic quadrant (as it ought to be). This discrepancy shows that the differing epicycles (stated by Āryabhaṭa) are incorrect.
>
> "Since the epicycle which is the multiplier of the Rsine of anomaly in the odd anomalistic quadrant is different from the epicycle which is the multiplier of the Rversine of anomaly in the even anomalistic quadrant, the (*manda* or *śīghra*) correction for the anomaly amounting to half a circle, does not vanish (as it ought to). This discrepancy, too, shows that differing epicycles (stated by Āryabhaṭa) are incorrect.[1]
>
> "Since the epicycles (stated by Āryabhaṭa) correspond to odd and even anomalistic quadrants (and not to their first or last points), the (so-called true) epicycle which is obtained by multiplying the Rsine of anomaly by the difference of the epicycles (for the odd and even quadrants) and dividing by the radius and then subtracting the resulting quotient from or adding that to the epicycle for the odd quadrant, according as it is greater or less than the other, is not the correct epicycle.
>
> "If indeed there should be two different epicycles for the odd and even anomalistic quadrants, then, why have not two

---

1. Let α and β be the epicycles for the odd and even quadrants and let the anomaly be equal to 180°. Then, according to Āryabhaṭa I (see *Ā*, iii. 22 (*a-b*)), the corresponding

$$bhuj\bar{a}phala = \alpha \times \text{R sine } 90^\circ/360 - \beta \times \text{Rvers } 90^\circ/360$$
$$= \alpha\ R/360 - \beta R/360,$$

which is not equal to zero, because $\alpha \neq \beta$.

different epicycles been stated in the case of the Sun and the Moon. It simply shows that the process of planetary correction stated in (Āryabhaṭa's) *Audayika-Tantra* (*i. e., Āryabhaṭīya*) does, in neither way, lead to a correct result."[1]

Had Āryabhaṭa I specified that the epicycles stated by him corresponded to the first or last points of the respective anomalistic quadrants, there would not have been any occasion for such a criticism.

The number 3375, denoting the length of the outer boundary of the terrestrial wind, has reminded Bhāskara I of the following formula which also involves that number :

$$\text{Rsin } \theta = \frac{4\,(180° - \theta)\,\theta.\,\text{R}}{12 \times 3375 - (180° - \theta)\theta},$$

where θ is in terms of degrees. Bhāskara I thinks that the length of the outer boundary of the terrestrial wind has been stated simply to teach the method of finding the Rsine without the use of the Rsine Table which is implied in the above formula.

Brahmagupta (A. D. 628) misreads *giyiṅaśa* as *giyigasa* and unnecessarily criticises Āryabhaṭa I for giving two different values of the Earth's diameter. Writes he :

> "The circumference being (stated as) 3393 *yojanas*, the Earth's diameter becomes equal to 1080 *yojanas*. By stating the same again as 1050 (*yojanas*) due to uncertainty of his mind, he (*i.e.*, Āryabhaṭa I) has exposed his knowledge !"[2]

---

1. औदयिको यः परिधिर्विषमेऽन्योऽन्यः समे भुजस्य गुणः ।
तदसद्विषमान्तफलं यतो न युग्मादिफलतुल्यम् ॥
विषमेऽन्योऽन्यो युग्मे परिधिर्गुणकः क्रमोत्क्रमज्यानाम् ।
चक्रार्धे फलनाशो न भवति यस्मादसत् तदपि ॥
व्यासार्धहृतो बाहुः परिधिविशेषाहतः फलोनयुतः ।
प्रथमोऽधिकोनको यत् तदसत् पदयोः परिधिपाठात् ॥
विषमसमयोर्यदि द्वौ परिधी किं सूर्यचन्द्रयोर्नोक्तौ ।
घटते न कथञ्चिदियं स्फुटक्रियौदयिकतन्त्रोक्ता ॥
BrSpSi, xi. 18-21

2. गसगियि योजनपरिधेः षञि भूव्यासः पुनर्विला वदता ।
आत्मज्ञानं ख्यापितमनिश्चयस्वमतिकृतकत्वात् ॥
BrSpSi, xi. 15

RSINE-DIFFERENCES

मखि भखि फखि धखि णखि ञखि
ङखि हस्झ स्ककि किष्ग श्घकि किघ्व[1] ।
घ्लकि किग्र हक्य धकि किच[2]
स्ग श्झ[3] ङ्व क्ल प्त फ छ कलार्धज्याः ॥ १२ ॥

12. 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, and 7— these are the Rsine-differences (at intervals of 225 minutes of arc) in terms of minutes of arc.

The following table gives the Rsines and the Rsine-differences at intervals of 225′ (or 3° 45′) according to Āryabhaṭa I and the corresponding modern values correct to three decimal places.

Table 10. Rsines and Rsine-differences at the intervals of 225′ or 3°45′

	Āryabhaṭa I's values		Modern Values	
Arc	Rsine	Rsine-differences	Rsine	Rsine-differences
225′	225′	225′	224′.856	224′.856
450′	449′	224′	448′.749	223′.893
675′	671′	222′	670′.720	221′.971
900′	890′	219′	889′.820	219′.100
1125′	1105′	215′	1105′.109	215′.289
1350′	1315′	210′	1315′.666	210′.557
1575′	1520′	205′	1520′.589	204′.923
1800′	1719′	199′	1719′.000	198′.411
2025′	1910′	191′	1910′.050	191′.050
2250′	2093′	183′	2092′.922	182′.872

1. D.G. Sū. किघ्वा ; others किघ्व.

2. A. हक्य घा कि कि च ; Bh. Śa. हक्य घकि किच ; E. हक्य घाह्रा किच. Pa. Ra. Sū. हक्य घाह्रा स्त ; So. किग्र क्यकि किघ किच.

3. Bh. भ्श ; others श्झ.

Arc	Āryabhaṭa I's values		Modern Values	
	Rsine	Rsine-differences	Rsine	Rsine-differences
2475′	2267′	174′	2266′.831	173′.909
2700′	2431′	164′	2431′.033	164′.202
2925′	2585′	154′	2584′.825	153′.792
3150′	2728′	143′	2727′.549	142′.724
3375′	2859′	131′	2858′.592	131′.043
3600′	2978′	119′	2977′.395	118′.803
3825′	3084′	106′	3083′.448	106′.053
4050′	3177′	93′	3176′.298	92′.850
4275′	3256′	79′	3255′.546	79′.248
4500′	3321′	65′	3320′.853	65′.307
4725′	3372′	51′	3371′.940	51′.087
4950′	3409′	37′	3408′.588	36′.648
5175′	3431′	22′	3430′.639	22′.051
5400′	3438′	7′	3438′.000	7′.361

The twenty-four Rsines given in the *Sūrya-siddhānta*[1] are exactly the same as those in column 2 above. P.C. Sengupta is of the opinion that the author of the *Sūrya-siddhānta* has based his Rsines on the Rsine-differences given by Āryabhaṭa I.[2]

The 16th Rsine, *viz.*, 2978, was modified by Āryabhaṭa II[3] (c. A.D. 950) who replaced it by the better value 2977. The table of Rsines given by Bhāskara II[4] (A.D. 1150) is the same as that of Āryabhaṭa II (c. A.D. 950).

Astronomer Sumati of Nepal, who lived anterior to Āryabhaṭa II (c. A.D. 950), gives[5] the values of the 4th and 16th Rsines as 889′ and 2977′ respectively instead of 890′ and 2978′ given by Āryabhata I. Sumati's table contains ninety Rsines at the intervals of one degree.

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1. ii. 17-22.

2. See P. C. Sengupta's introduction (p. xix) to E. Burgess' *Translation of the Sūrya-siddhānta.*

3. See *MSi,* iii. 4-6.

4. See *SiŚi, Grahagaṇita,* ii. 3-6.

5. Both in *Sumati-mahātantra* and *Sumati-karaṇa.*

AIM OF THE DAŚAGĪTIKĀ-SŪTRA

दशगीतिकसूत्रमिदं[1] भूग्रहचरितं भपञ्जरे ज्ञात्वा ।
ग्रहभगणपरिभ्रमणं[2] स याति भित्त्वा परं ब्रह्म ॥ १३ ॥

[इति गीतिकापादः समाप्तः][3]

13. Knowing this *Daśagītikā-sūtra*, (giving) the motion of the Earth and the planets, on the Celestial Sphere (Sphere of asterisms or Bhagola), one attains the Supreme Brahman after piercing through the orbits of the planets and stars.

This chapter is called 'Ten Aphorisms in *Gītikā* Stanzas' (*Daśagītikāsūtra*). But instead of 10 *gītikā* stanzas there are 11 *gītikā* stanzas (vss. 2-12) here. The question arises : Which of these are those 10 which contain the 10 aphorisms of this chapter ? This is indeed a controversial question. For, according to the commentators Bhāskara I (A.D. 629), Someśvara and Sūryadeva (b. A.D. 1191), vss. 2-11 are the ten stanzas which contain the 10 aphorisms ; vs. 12, in their opinion, does not constitute an aphorism as it contains a table of Rsine-differences which is easily derivable. According to the commentator Parameśvara (A.D. 1431), however, vss, 3-12 are the 10 stanzas containing the 10 aphorisms; vs. 2, in his opinion, is a definition and not a mathematical aphorism.

There is, however, another difficulty. Is vs. 12 composed in the *gītikā* metre or in the *āryā* metre ? According to Sūryadeva, it is in the *āryā* metre and, according to Parameśvara, it is in the *gītikā* metre. In fact, vs. 12 (in the form in which Sūryadeva and Parameśvara state it) is, as pointed out by H. Kern,[4] metrically defective, as it contains 20 syllabic instants instead of 18, in the fourth quarter :

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1. C.D.E. Kṛ. Ra. Sū. दशगीतिसूत्रमेतद् ।

2. Kṛ. ग्रहगोलपरिभ्रमणं ।

3. A. दशगीतिकासूत्रं समाप्तम् ; B.D. No colophon ; E. गीतिसूत्रं समाप्तम् ।

4. See H. Kern, *Āryabhaṭīyam*, Leiden (1874), p. 17, footnote.

I I I I I I I I I I I I I I S S I I S S I I S S
मखि भखि फखि धखि णखि ञखि, ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।

I I S I S I S S S S S S S I I I I S S S
घ्लकि किग्र ह्वय धाह्रा, स्त स्ग श्झ ङ्व क्ल प्त फ छ कलार्धज्याः ॥

It may be called a defective *gītikā*. It is not a pure *āryā*. However, in the forms in which vs. 12 has been stated by Bhāskara I and Someśvara, it is in the perfect *gītikā* metre.

Bhāskara I's reading

I I I I I I I I I I I I I I I S S I I S S I I S S
मखि भखि फखि धखि णखि ञखि, ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।

I I S I S I I I I I I I S S S I I I I S S S
घ्लकि किग्र हक्य धकि किच, स्ग श्झ ङ्व क्ल प्त फ छ कलार्धज्याः ।

Someśvara's reading

I I I I I I I I I I I I I I S S I I S S I I S S
मखि भखि फखि धखि णखि ञखि, ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।

I I S S I I I I I I S S S S I I I I S S S
घ्लकि किग्र वधकि किघ किच, स्ग श्झ ङ्व क्ल प्त फ छ कलार्धज्याः ॥

We agree with Parameśvara in regarding vss. 3 to 12 as forming the 10 *gītika* stanzas containing the 10 aphorisms of this chapter.

CHAPTER II

GAṆITA OR MATHEMATICS

INVOCATION AND INTRODUCTION

ब्रह्म-कु-शशि-बुध-भृगु-रवि-
कुज-गुरु-कोण-भगणान् नमस्कृत्य ।
आर्यभटस्त्विह निगदति
कुसुमपुरेऽभ्यर्चितं ज्ञानम्[1] ॥ १ ॥

1. **Having bowed with reverence to Brahmā, Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn and the asterisms, Āryabhaṭa sets forth here the knowledge honoured at Kusumapura.**

Commenting on this stanza, Bhāskara I writes : "Kusumapura is Pāṭaliputra. (Āryabhaṭa) sets forth the knowledge honoured there. This is what one hears said : Indeed this *Svāyambhuva-siddhānta* was honoured by the learned people of Kusumapura (Pāṭaliputra), although the *Pauliśa-*, *Romaka-*, *Vāsiṣṭha-* and *Saurya-Siddhāntas* were also (known) there. That is why (Āryabhaṭa) says—'the knowledge honoured at Kusumapura'."

THE FIRST TEN NOTATIONAL PLACES

एकं[2] दश च शतं च सहस्रं त्वयुत[3]नियुते तथा प्रयुतम् ।
कोट्यर्बुदं च वृन्दं स्थानात् स्थानं दशगुणं स्यात् ॥ २ ॥

2. *Eka* **(units place),** *daśa* **(tens place),** *śata* **(hundreds place),** *sahasra* **(thousands place),** *ayuta* **(ten thousands place),** *niyuta* **(hundred thousands place),** *prayuta* **(millions place),** *koṭi* **(ten millions place),** *arbuda* **(hundred millions place),**

1. C. शास्त्रम्
2. Bh. adds च
3. A-G. Gh. Nī. Pa. Ra. So. Sū. सहस्रमयुत

and *vṛnda* **(thousand millions place) are, respectively, from place to place, each ten times the preceding.**[1]

The notational places are denoted by writing zeros as follows :

0 0 0 0 0 0 0 0 0 0

The zero on the extreme right denotes the units place, the next one (on its left) denotes the tens place, the next one denotes the hundreds place, and so on.

SQUARE AND SQUARING

वर्गः समचतुरश्रः फलं च सदृशद्वयस्य संवर्गः ।

3. (a-b) **An equilateral quadrilateral with equal diagonals and also the area thereof are called 'square'. The product of two equal quantities is also 'square'.**[2]

The commentator Parameśvara explains the term *samacaturaśra* as follows : "That four-sided figure whose four sides are equal to one another and whose two diagonals are also equal to each other is called a *samacaturaśra.*"

By defining a square as the product of two equal quantities the author has stated, by implication, the rule of squaring. That is, to find the square of a number, one should multiply that number by itself.

The commentator Bhāskara I gives the terms *varga, karaṇī, kṛti, vargaṇā*[3] and *yāvakaraṇa* as synonyms, meaning 'square or squaring'. Of these terms, *karaṇī, vargaṇā* and *yāvakaraṇa* are unusual. The term *yāvakaraṇa* is derived from the fact that in Hindu algebra x^2 is written as *yāva* (*yā* standing for *yāvat-tāvat, i.e., x,* and *va* for *varga, i.e.,* square).[4]

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1. *Cf. GSS*, i. 63-68 ; *PG*, def. 7-8 ; *GT*, p. 1, vv. 2-3 ; *L* (ASS), Def. 10-11, pp. 11-12 ; *GK*, I, p. 1, vv. 2-3.

2. *Cf. BrSpSi*, xviii. 42 ; *GSS*, ii. 29 ; *SiŚe*, xiii. 4 ; *L* (ASS), Rule 19 (a), p. 19.

3. Use of the term *vargaṇā* in the sense of multiplication has been made by Bhaṭṭotpala also. See his comm. on *BṛJā*, vii. 13.

4. See vol. II, Introduction, p. lxxvii, sec. 4.

The terms for multiplication according to Bhāskara I are: *saṁvarga, ghāta, guṇanā, hatiḥ* and *udvartanā*. For the multiplication of equal quantities, Bhāskara I uses a special term, *gata*, meaning literally 'moved' > progressed > raised. "*Guṇanā* is the multiplication (*abhyāsa*) of unequal quantities, and *gata*", says he, "is the multiplication of equal quantities."[1] The term *dvigata*, according to him, means square, *trigata* means 'cube'; and so on. The *dvigata* of 4 is the product of 4 and 4, *i.e.*, 4^2; the *trigata* of 4 is the product of 4 and 4 and 4, *i.e.*, 4^3; and so on. According to this terminology, m^n will be expressed by saying '*n*th *gata* of *m*', which corresponds to our present-day expression '*n*th power of *m*'. Following the same terminology, the roots have been called *gatamūla*. Thus 4 is the *gatamūla* of 4^2, the *trigatamūla* of 4^3, and so on. In general, *m* is the '*n*th *gatamūla* of m^n'. This, too, corresponds to our present-day expression 'the *n*th root of m^n'.

It is interesting to note that Bhāskara I finds fault with the usual Hindu method[2] of squaring a number for the simple reason that it implies the use of the squares of the digits 1 to 9 but it neither states them nor gives the method for obtaining them. Āryabhaṭa I's method, according to him, is complete in itself.

CUBE AND CUBING

सदृशत्रयसंवर्गो घनस्तथा द्वादशाश्रिः स्यात्[3] ॥ ३ ॥

3. (c-d) The continued product of three equals as also the (rectangular) solid having twelve (equal) edges is called a 'cube'.[4]

The rule for cubing a number is implied as in the previous case.

Here also, Bhāskara I finds fault with the usual Hindu method[5] of cubing a number for the reason that although it implies the use of

---

1. See vol. II, Bhāskara I's commentary, p. 43.
2. For example, see *PG*, Rule 23.
3. F. G. द्वादशाश्रिश्च ; Pa. Ra. द्वादशाश्रस्स्यात्
4. *Cf. BrSpSi*, xviii. 42 ; *GSS*, ii. 43 ; *SiŚe*, xiii. 4 ; *L* (ASS), Rule 24 (a), p. 23.
5. For example, see *PG*, Rule 27-28.

the cubes of the digits 1 to 9, it neither states them nor tells how to find them out.

SQUARE ROOT

भागं हरेदवर्गान्नित्यं द्विगुणेन वर्गमूलेन ।
वर्गाद्वर्गे शुद्धे लब्धं स्थानान्तरे मूलम् ॥ ४ ॥

4. **(Having subtracted the greatest possible square from the last odd place and then having written down the square root of the number subtracted in the line of the square root) always divide[1] the even place (standing on the right) by twice the square root. Then, having subtracted the square (of the quotient) from the odd place (standing on the right), set down the quotient at the next place (*i.e.*, on the right of the number already written in the line of the square root). This is the square root. (Repeat the process if there are still digits on the right).[2]**

The following example will illustrate the above rule.

Example. Find the square root of 55,225.

Let the odd and even places be denoted by *o* and *e*, respectively. The various steps are then as shown below :

		235
	o e o e o	line of square root
	5 5 2 2 5	
Subtract square	4	
Divide by twice the root	4) 1 5 (3	
	1 2	
	3 2	

1. In dividing, the quotient should be taken as great as will allow the subtraction of its square from the next odd place.

2. *Cf. GSS*, ii. 36 ; *PG*, Rule 25-26 ; *GT*, p. 9, vs. 23 ; *MSi*, xv. 6 (*c-d*)-7 ; *SiŚe*, xiii. 5 ; *L* (ASS), p. 21, Rule 22 ; *GK*, I, p. 7, lines 2-9.

```
                                    3 2
Subtract square of quotient           9
                                   --------
Divide by twice the root       46) 2 3 2 (5
                                   2 3 0
                                   --------
                                       2 5
Subtract square of quotient            2 5
                                      -----
                                         0
```

The process ends. The square root is 235. The remainder being zero, the square root is exact.

G.R. Kaye's statement that Āryabhaṭa I's method is algebraic in character and that it resembles the method given by Theon of Alexandria, are, as noted by W.E. Clark, B. Datta and A.N. Singh,[1] incorrect.

CUBE ROOT

अघनाद् भजेद् द्वितीयात्
त्रिगुणेन घनस्य मूलवर्गेण ।
वर्गस्त्रिपूर्वगुणितः
शोध्यः प्रथमाद् घनश्च घनात् ॥ ५ ॥

5. **(Having subtracted the greatest possible cube from the last cube place and then having written down the cube root of the number subtracted in the line of the cube root), divide the second non-cube place (standing on the right of the last cube place) by thrice the square of the cube root (already obtained) ; (then) subtract from the first non-cube place (standing on the right of the second non-cube place) the square of the quotient multiplied by thrice the previous (cube root) ; and (then subtract) the cube (of the quotient) from the cube place (standing on the right of the first non-cube place) (and write down the quotient on the right of the previous cube root in the line of the cube root, and treat this as the new cube root. Repeat the process if there are still digits on the right).[2]**

---

1. See Datta and Singh, *History of Hindu mathematics*, Part I, p. 171. For details see A. N. Singh, *BCMS*, 18 (1927). See also W. E. Clark, *Āryabhaṭīya*, pp. 23 f.

2. *Cf. BrSpSi*, xii. 7 ; *GSS*, ii. 53-54 ; *PG*, Rule 29-31 ; *MSi*, xv. 9-10 (*a-b*) ; *GT*, p. 13, lines 18-25 ; *SiŚe*, xiii. 6-7 ; *L* (ASS), Rule 28-29, pp. 27-28 ; *GK*, I, pp. 8-9, vv. 24-25.

Beginning from the units place, the notational places are called cube place, first non-cube place, second non-cube place, cube place, first non-cube place, second non-cube place, cube place, and so on. Indicating the cube, first non-cube and second non-cube places by c, n and n', their positions may be shown as below :

$$c \; n' \; n \; c \; n' \; n \; c \; n' \; n \; c \; n' \; n \; c$$

$$0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0$$

The following solved example will explain the rule stated in the above stanza :

Example. Find the cube root of 17,71,561

```
                        c n' n  c n' n  c              121
                        1  7 7  1  5 6  1        line of cube root

Subtract 1³               1
Divide by 3.1²         3) 0 7 (2
                            6
                            1 7
Subtract 3.1.2²             1 2
                              5 1
Subtract 2³                     8
Divide by 3.12²        432) 4  3  5 (1
                            4  3  2
                                  3 6
Subtract 3.12.1²                  3 6
                                    0 1
Subtract 1³                           1
                                      0
```

The process ends. The required cube root is 121. The remainder being zero, the root is exact.

AREA OF A TRIANGLE

त्रिभुजस्य फलशरीरं समदलकोटीभुजार्धसंवर्गः ।

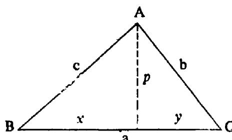
6. (a-b) **The product of the perpendicular (dropped from the vertex on the base) and half the base gives the measure of the area of a triangle.**

The term *samadalakoṭī* means 'the perpendicular dropped from the vertex on the base of a triangle', *i.e.*, 'the altitude of a triangle'. Bhāskara I criticises those who interpret it as meaning 'the upright which bisects the triangle into two equal parts', for, in that case, the above rule will be applicable only to equilateral and isosceles triangles.

The word *phalaśarīra* means, according to Bhāskara I, *phala-pramāṇa*, *i.e.*, 'the measure or amount of the area'.

The above rule is applicable when the base and the altitude of a triangle are known. When the three sides of a triangle are given but the altitude is not known, Bhāskara I gives the following formulae to get the segments of the base (called *ābādhā* or *ābādhāntara*) and the altitude :

Fig. 1



(1) $x=\frac{1}{2}\left(a+\frac{c^2-b^2}{a}\right)$

(2) $y=\frac{1}{2}\left(a-\frac{c^2-b^2}{a}\right)$

(3) $p=\sqrt{c^2-x^2}$ or $\sqrt{b^2-y^2}$

It is remarkable that Bhāskara I does not mention the formula :

Area of a triangle $=\sqrt{s(s-a)(s-b)(s-c)}$, $2s=a+b+c$, although his contemporary Brahmagupta states it in his *Brāhma-sphuṭa-siddhānta*.[1]

VOLUME OF RIGHT PYRAMIDS

ऊर्ध्वभुजातत्संवर्गार्धं[2] स घनः षडश्रिरिति ॥ ६ ॥

6. (c-d) Half the product of that area (of the triangular base) and the height is the volume of a six-edged solid.

---

1. See *BrSpSi*, xii. 21.
2. A. B. F. संवर्गार्धः

This rule, which is based on speculation on the analogy of the area of a triangle, is inaccurate. The correct formula is found to occur in the *Brāhma-sphuṭa-siddhānta* of Brahmagupta where it is stated as follows :

> "The volume of a uniform excavation divided by three is the volume of the needle-shaped solid."[1]

That is to say,

$$\text{Volume of a cone or pyramid} = \tfrac{1}{3}\,(\text{area of base}) \times (\text{height}).$$

Bhāskara I seems to be unaware of this formula, for he has no comment to make on the rule of Āryabhaṭa I. Even the commentators Someśvara and Sūryadeva (*b.* A.D. 1191) have nothing to add.

AREA OF A CIRCLE

समपरिणाहस्यार्धं विष्कम्भार्धहतमेव वृत्तफलम् ।

7. (a-b) Half of the circumference, multiplied by the semi-diameter certainly gives the area of a circle.

That is,

$$\text{area of a circle} = \tfrac{1}{2} \times \text{circumference} \times \text{radius}.$$

The same result in the form

$$\text{area of a circle} = \frac{\text{circumference} \times \text{diameter}}{4}$$

occurs earlier in the *Tattvārthādhigama-sūtra-bhāṣya*[2] of Umāsvāti (1st century A.D.). It occurs in the *Bṛhat-kṣetra-samāsa*[3] of Jinabhadra Gaṇi (A.D. 609) also.

VOLUME OF A SPHERE

तन्निजमूलेन हतं घनगोलफलं निरवशेषम् ॥ ७ ॥

7. (c-d) That area (of the diametral section) multiplied by its own square root gives the exact volume of a sphere.

---

1. See *BrSpSi*, xii. 44.
2. Comm. iii. 11.
3. i. 7.

That is, if r be the radius of a sphere, then, according to Āryabhaṭa I :

$$\text{Volume of a sphere} = \pi r^2 \sqrt{\pi r^2}.$$

This formula is based on speculation, and, as noted by Bhāskara I, is inaccurate, although called exact by Āryabhaṭa I.

The probable rationale of Āryabhaṭa I's formula is as follows :

The area of a circle of radius r

$= \pi r^2$

$=$ area of a square of side $\sqrt{\pi r^2}$. (*vide* vs. 9 *a-b*)

On the analogy of this, Āryabhaṭa I concludes that

Volume of a sphere of radius r

$=$ volume of a cube of edge $\sqrt{\pi r^2}$

$= \sqrt{\pi r^2} \times \sqrt{\pi r^2} \times \sqrt{\pi r^2}$

$= \pi r^2 \times \sqrt{\pi r^2}$.

Bhāskara I quotes the following formula from some earlier work, but he does not give it any credit and regards it as inferior to that given by Āryabhaṭa I :

$$\text{Volume of a sphere of radius } r = \tfrac{9}{2} r^3.$$

It is noteworthy that Bhāskara I's contemporary Brahmagupta, who has criticised Āryabhaṭa I even for his minutest errors, has not been able to make any improvement on Āryabhaṭa's formula for the volume of a sphere. Still more noteworthy is the fact that mathematicians and astronomers in northern India, too, regarded Āryabhaṭa I's formula as accurate and went on using it even in the second half of the ninth century A.D. Brahmagupta's commentator Pṛthūdaka who wrote his commentary on the *Brāhma-sphuṭa-siddhānta* in 860 A.D. at Kannauj, has prescribed[1] Āryabhaṭa I's rule for finding the volume of a sphere.

The formulae given by other Indian mathematicians are :

(1) Mahāvīra's (850 A.D.) formula :[2]

$$\text{Volume of a sphere} = \tfrac{9}{2} \times \tfrac{9}{10} r^3.$$

---

1. In his comm. on *BrSpSi*, xi. 20.

2. See *GSS*, viii. 28½.

(2) Śrīdhara's (*c.* 900 A.D.) formula :[1]

$$\text{Volume of a sphere} = 4(1 + 1/18)r^3.$$

The same formula is given by Āryabhaṭa II (*c.* 950 A.D.)[2] and Śrīpati (1039 A.D.).[3]

All these formulae are approximate. The accurate formula was given by Bhāskara II (1150 A.D.).

(3) Bhāskara II's (1150 A.D.) accurate formula :[4]

$$\text{Volume of a sphere} = \tfrac{1}{6} \times \text{surface} \times \text{diameter}.$$

Bhāskara II also gave the following approximate formula, using $\pi = 22/7$:

$$\text{Volume of a sphere} = \frac{(\text{diameter})^3}{2}(1 + 1/21)$$

$$\text{or } 4\,(1 + 1/21)\, r^3 \text{ approx.}$$[5]

AREA OF A TRAPEZIUM

आयामगुणे पार्श्वे तद्योगहृते स्वपातरेखे ते ।
विस्तरयोगार्धगुणे ज्ञेयं क्षेत्रफलमायामे ॥ ८ ॥

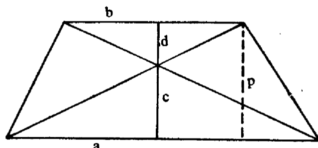
8. **(Severally) multiply the base and the face (of the trapezium) by the height, and divide (each product) by the sum of the base and the face : the results are the lengths of the perpendiculars on the base and the face (from the point of intersection of the diagonals). The results obtained by multiplying half the sum of the base and the face by the height is to be known as the area (of the trapezium).**

---

1. See *Triś*, p. 39, Rule 56.
2. See *MSi*, xv. 108.
3. See *SiŚe*, xiii. 46.
4. See *L* (Ānandāśrama), Rule 201 (*c-d*), p. 201.
5. See *L* (Ānandāśrama), Rule 203 (*e-f*), p. 203.

Let a, b be the base and the face, p the height and c, d the lengths of the perpendiculars on the base and the face from the point where the diagonals intersect. Then

Fig. 2



$$c = \frac{ap}{a+b}$$

$$d = \frac{bp}{a+b}$$

$$\text{area} = \tfrac{1}{2}(a+b)\,p.$$

The term *āyāma,* meaning 'breadth', denotes the height of the trapezium. The term *vistara,* meaning 'length', denotes the base and face of the trapezium and so *vistarayogārdha* means 'half the sum of the base and the face'.

The term *pārśve* means, here, the two sides of a trapezium lying on the two sides of the height. Evidently, they are the base and the face.

AREA OF PLANE FIGURES

सर्वेषां क्षेत्राणां प्रसाध्य पार्श्वे फलं तदभ्यासः ।

9. (a-b) In the case of all the plane figures, one should determine the adjacent sides (of the rectangle into which that figure can be transformed) and find the area by taking their product.

According to Bhāskara I, this rule is meant both for finding the area and for verifying the area of a plane figure. Writes he :

Doubt : "Now, the word *all* means 'everything without exception'; so, here, all (plane) figures are included. The area of all (plane) figures being thus determined by this rule, the statement of the previously stated rules becomes useless.

Answer : That is not useless. Both the verification and the calculation of the areas are taught by this rule. The areas of the previously stated figures have to be verified. The mathematicians Maskarī, Pūraṇa and Pūtana etc., prescribe the verification of all (plane) figures (by transforming them) into a rectangular figure. So has it been said :

> 'Having determined the area in accordance with the prescribed rule, verification should always be made by (transforming the plane figure into) a rectangle, because it is only of the rectangle that the area is obvious.'
>
> 'The determination of the area of the (plane) figures which have not been mentioned above is possible only by transforming them into rectangles.'

The commentator Someśvara, following Bhāskara I, is of the opinion that the above rule is meant for the verification of the plane figures. According to the commentators Sūryadeva (*b*. 1191 A.D.), Parameśvara (1431 A.D.), Yallaya (1480 A.D.), and Raghunātha-rāja (1597 A.D.), this rule is meant for finding the area of all plane figures including those already considered above. According to the commentator Nīlakaṇṭha (*c*. 1500 A.D), however, this rule is meant only for finding the area of those plane figures that have not been considered heretofore.

There is, however, no doubt that the above rule is based on the assumption that all plane figures can be transformed into a rectangle.

In his commentary, Bhāskara I has shown how to find the area of a triangle, a quadrilateral, a drum-shaped figure, and a figure resembling the tusk of an elephant, by transforming them into rectangles.

CHORD OF ONE-SIXTH CIRCLE

परिधेः षड्भागज्या विष्कम्भार्धेन सा तुल्या ॥ ९ ॥

9. (c-d) The chord of one-sixth of the circumference (of a circle) is equal to the radius.[1]

---

1. *Cf. PSi*, iv. 2 (*a-b*).

That is,

$$\text{chord } 60^\circ = R,$$
$$\text{or} \quad R\sin 30^\circ = R/2.$$

CIRCUMFERENCE-DIAMETER RATIO

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥ १० ॥

10. 100 plus 4, multiplied by 8, and added to 62,000 : this is the nearly approximate measure of the circumference of a circle whose diameter is 20,000.

This gives

$$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{62832}{20000} = 3{\cdot}1416.$$

This value does not occur in any earlier work on mathematics, and forms an important contribution of Āryabhaṭa I.

It is noteworthy that Āryabhaṭa I has called the above value approximate.

COMPUTATION OF RSINE-TABLE GEOMETRICALLY

समवृत्तपरिधिपादं छिन्द्यात् त्रिभुजाच्चतुर्भुजाच्चैव ।
समचापज्यार्धानि तु विष्कम्भार्धे यथेष्टानि ॥ ११ ॥

11. Divide a quadrant of the circumferenc of a circle (into as many parts as desired). Then, from (right) triangles and quadri. laterals, one can find as many Rsines of equal arcs as one likes, for any given radius.

Following Bhāskara I, we explain the method implied in the above stanza by solving three examples.

Example 1. Find six Rsines at intervals of 15° in a circle of radius 3438′.

Let Fig. 3 represent a circle of radius R (=3438′). Divide its circumference into twelve equal parts by the points A, B, C, D, E, F, ..., L. Join BL. This is equal to R and denotes chord 60°. Half of this, *i.e.*, MB, is Rsin 30°. Thus Rsin 30°=R/2=1719′. This is the second Rsine.

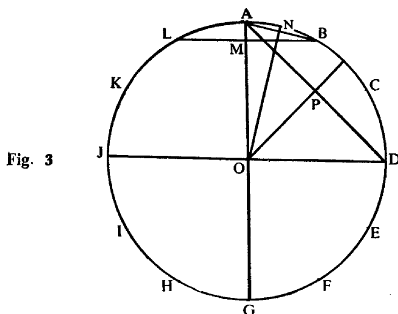


Fig. 3

Now, in the right-angled triangle OMB,

$$OM=\sqrt{R^2-(R/2)^2}=\frac{\sqrt{3}}{2}R=2978'.$$

This is the fourth Rsine, *viz.,* Rsin 60°.

Now, in the right-angled triangle AMB, AM=Rvers 30° and MB=Rsin 30°.

$$\therefore AB=\sqrt{(\text{Rsin } 30^\circ)^2+(\text{Rvers } 30^\circ)^2}$$

This is chord 30°. Half of this (*i.e.,* AN) is Rsin 15°. Thus

$$\text{Rsin } 15^\circ=\tfrac{1}{2}\sqrt{(\text{Rsin } 30^\circ)^2+(\text{Rvers } 30^\circ)^2}$$
$$=890'.$$

This is the first Rsine.

Now, in the triangle ANO, AN=Rsin 15° and OA=R.

$\therefore ON=\sqrt{R^2-(\text{Rsin } 15^\circ)^2}=3321'$. This is the fifth Rsine, *i.e.,* Rsin 75°.

Since this is the fifth Rsine, *i.e.,* an odd Rsine, it would not yield any further Rsine.

Thus, five Rsines have been obtained by using triangles. Now, we shall make use of the semi-square AOD, whose sides OA and OD are each equal to R. Therefore $AD=\sqrt{2}\,R$. This is chord 90°. Half

of this (*i.e.*, AP) is Rsin 45°. Thus, Rsin $45° = R/\sqrt{2} = 2431'$. This is the third Rsine.

Thus we get all the six Rsines, which are as follows :

Rsin $15° = 890'$; Rsin $30° = 1719'$; Rsin $45° = 2431'$;
Rsin $60° = 2978'$; Rsin $75° = 3321'$; Rsin $90° = 3438'$.

Analysis. Verse 9 (c-d) gives the second Rsine. This yields the first and the fourth Rsines. The first Rsine yields the fifth Rsine. The fourth and the fifth Rsines do not yield any other Rsines. This process ends here.

Again, the radius is the sixth Rsine. It yields the third Rsine. The third Rsine being odd, does not yield and further Rsine. So this process also ends.

Thus, from the second and the sixth Rsines, one gets all the six desired Rsines.

Example 2. Find twelve Rsines at intervals of 7° 30′ in the circle of radius R (=3438′).

Fig. 4 represents a circle of radius R (3438′). Join LB, as before. This is equal to R and denotes chord 60°. Half of this is Rsin 30°. Thus,

Fig. 4

$$\text{Rsin } 30° = R/2 = 1719'.$$

This is the fourth Rsine.

Now, from the right-angled triangle OMB, as before,

$$OM=\sqrt{R^2-(R/2)^2}=\frac{\sqrt{3}}{2}R=2978'.$$

This is Rsin 60°, *i e.*, the eighth Rsine.

Now, from the right-angled triangle AMB,

$$AB=\sqrt{(\text{Rsin } 30°)^2+(\text{Rvers } 30°)^2}$$
$$=\sqrt{(1719')^2+(460')^2}=1780'.$$

This is chord 30°. Half of this, *i.e.*, AN, is Rsin 15°. Thus,

$$\text{Rsin } 15°=890'.$$

This is the second Rsine.

Now from the right-angled triangle ANO

$$ON=\sqrt{(AO)^2-(AN)^2}=\sqrt{R^2-(\text{Rsin } 15°)^2}=3321'.$$

This is Rsin 75°, *i e.*, the tenth Rsine.

Now, from the right-angled triangle ANR, where R is the mid-point of the arc AB, we have

$$AR=\sqrt{(AN)^2+(NR)^2}=\sqrt{(\text{Rsin } 15°)^2+(\text{Rvers } 15°)^2}$$
$$=\sqrt{(890')^2+(117')^2}=898'.$$

This is chord 15°. Half of this (*i.e.*, AS) is Rsin 7° 30′. Thus,

$$\text{Rsin } 7° 30'=449'.$$

This is the first Rsine.

Now, from the right-angled triangle ASO,

$$OS=\sqrt{R^2-(\text{Rsin } 7° 30')^2}=3409'.$$

This is Rsin (82° 30′), *i.e.*, the eleventh Rsine.

Now, Rvers 75° = R—Rsin 15°, so that

$$\text{chord } 75°=\sqrt{(\text{Rsin } 75°)^2+(\text{Rvers } 75°)^2}=4186'.$$

Half of this is Rsin 37° 30′. This is the fifth Rsine.

Now, $\text{Rsin } 52° 30'=\sqrt{R^2-(\text{Rsin } 37° 30')^2}=2728'$.

This is the seventh Rsine,

Thus, seven Rsines have been obtained by using triangles.

Now, we make use of the semisquare AOD as before. Its side OA and OD are each equal to R. Therefore,

$$AD=\sqrt{2}\,R=4862'.$$

This is chord 90°. Half of this, *i.e.*, AP, is Rsin 45°. Thus, Rsin 45° $=2431'$. This is the sixth Rsine.

Now, from the right-angled triangle APT,

$$AT=\sqrt{(\text{Rsin }45°)^2+(\text{Rvers }45°)^2}=2630'.$$

This is chord 45°. Half of this is Rsin 22° 30′. This is the third Rsine.

Hence, as before,

$$\text{Rsin }67°\ 30'=\sqrt{R^2-(\text{Rsin }22°\ 30')^2}=3177'.$$

This is the ninth Rsine.

Thus, we get all the twelve Rsines, which might be set out as follows :

Rsin 7° 30′=449′	Rsin 37° 30′=2093′	Rsin 67° 30′=3177′
Rsin 15°=890′	Rsin 45°=2431′	Rsin 75°=3321′
Rsin 22° 30′=1315′	Rsin 52° 30′=2728′	Rsin 82° 30′=3409′
Rsin 30°=1719′	Rsin 60°=2978′	Rsin 90°=3438′

Analysis. Stanza 9 (c-d) gives the fourth Rsine. This fourth Rsine yields the eighth and the second Rsines. The eighth Rsine does not yield any new Rsine. The second Rsine yields the tenth and the first Rsines. The first Rsine yields the eleventh Rsine, and the tenth Rsine yields the fifth and the seventh Rsines. These Rsines do not yield any new Rsines. So this process ends here.

Again, the radius is the twelfth Rsine. This yields the sixth Rsine, and the sixth Rsine yields the third and the ninth Rsines. These do not yield any further Rsines. So the process ends here.

Thus, from the fourth and the twelfth Rsines one gets all the twelve desired Rsines.

Example 3. Find the twentyfour Rsines at the equal intervals of 3° 45′ in the circle of radius R (=3438′).

From stanza 9 (*c-d*)

8th Rsine$=R/2=1719'$.

This yields :

16th Rsine$=\sqrt{R^2-(R/2)^2}=2978'$

4th Rsine$=\frac{1}{2}\sqrt{(R\sin 30^\circ)^2+(R\text{vers } 30^\circ)^2}$

$=890'$.

The 16th Rsine does not yield and new Rsine. The 4th Rsine yields :

20th Rsine$=\sqrt{R^2-(890')^2}=3321'$

2nd Rsine $=449'$

The 20th Rsine yields :

10th Rsine$=2093'$

The 2nd Rsine yields :

1st Rsine$=225'$

22nd Rsine$=3409'$

The 1st Rsine yields :

23rd Rsine$=3431'$

and the 22nd Rsine yields :

11th Rsine$=2267'$.

The 23rd Rsine does not yield any new Rsine.

The 11th Rsine yields :

13th Rsine$=2585'$.

The 13th Rsine does not yield any new Rsine.

The 10th Rsine yields :

5th Rsine$=1105'$

14th Rsine$=2728'$.

The 5th Rsine yields :

19th Rsine$=3256'$.

The 14th Rsine yields :

7th Rsine$=1520'$.

The 7th Rsine yields :

17th Rsine$=3084'$.

The 17th Rsine does not yield any new Rsine.

Now, we start with :

24th Rsine = 3438′.

This yields :

12th Rsine = 2431′.

This 12th Rsine yields :

6th Rsine = 1315′.

The 6th Rsine yields :

3rd Rsine = 671′

18th Rsine = 3177′.

The 3rd Rsine yields :

21st Rsine = 3372′

and the 18th Rsine yields :

9th Rsine = 1910′.

The 9th Rsine yields :

15th Rsine = 2859′.

Thus, we get all the twentyfour Rsines.

DERIVATION OF RSINE-DIFFERENCES

प्रथमाच्चापज्यार्धाद्यैरूनं खण्डितं द्वितीयार्धम् ।
तत्प्रथमज्यार्धांशैस्तैस्तैरूनानि शेषाणि ॥ १२ ॥

12. **The first Rsine divided by itself and then diminished by the quotient gives the second Rsine-difference. The same first Rsine diminished by the quotients obtained by dividing each of the preceding Rsines by the first Rsine gives the remaining Rsine-differences.**

Let R_1, R_2, ...R_{24} denote the twentyfour Rsines and δ_1 $(=R_1)$, δ_2, δ_3, ... , δ_{24} denote the twentyfour Rsine-differences. Then, according to the above rule,

$$\left.\begin{aligned} \delta_2 &= R_1 - \frac{R_1}{R_1} \\ \delta_{n+1} &= R_1 - \frac{R_1+R_2+\ldots+R_n}{R_1} \end{aligned}\right\} \quad (1)$$

The above translation is based on Prabhākara's interpretation of the text. The same interpretation is given by the commentators Someśvara, Sūryadeva (b. 1191 A. D.), Yallaya (1480 A. D.) and Raghunātha-rāja (1597 A.D.). It is interesting to note that this interpretation is also in agreement with the rule stated in the *Sūrya-siddhānta* (ii. 15-16), as interpreted by the commentator Raṅganātha (1603 A.D.), *viz.*.

$$R_{n+1}=R_n+R_1 - \frac{R_1+R_2+\ldots+R_n}{R_1}.$$

Datta and Singh, following the commentator Parameśvara (1431 A.D.), have translated the text as follows :

> "The first Rsine divided by itself and then diminished by the quotient will give the second difference. For computing any other difference, (the sum of) all the preceding differences is divided by the first Rsine and the quotient is subtracted from the preceding difference. Thus, all the remaining differences (can be calculated)."[1]

That is

$$\left.\begin{aligned} \delta_2 &= R_1 - \frac{R_1}{R_1} \\ \delta_{n+1} &= \delta_n - \frac{\delta_1+\delta_2+\ldots+\delta_n}{R_1} \\ &\text{or } \delta_n - \frac{R_n}{R_1}. \end{aligned}\right\} \quad (2)$$

This is also how the commentator Someśvara seems to have interpreted the text.

One can easily see that (1) and (2) are equivalent.

The commentator Nīlakaṇṭha (*c.* 1500 A.D.) interprets the text as follows :

> "The first Rsine divided by itself and then diminished by the quotient gives the second Rsine-difference. To obtain any other

1. *History of Hindu mathematics*, Part III, (unpublished). Also see A.N. Singh, 'Hindu Trigonometry', *Proc. Beṇaras Math. Soc.*, vol. 1, N.S., 1939, p. 88.

Rsine-difference, divide the preceding Rsine by the first Rsine and multiply the quotient by the difference between the first and second Rsine-differences and subtract the resulting product from the preceding Rsine-difference."

That is,

$$\left.\begin{aligned} \delta_2 &= R_1 - \frac{R_1}{R_1} \\ \delta_{n+1} &= \delta_n - (R_n/R_1)(\delta_1 - \delta_2). \end{aligned}\right\} \quad (3)$$

This is the accurate form of the formula, and reduces to the previous form because, according to Āryabhaṭa I,

$$\delta_1 - \delta_2 = 225 - 224 = 1.$$

The following is the trigonometrical rationale of (3) :

$$\delta_n - \delta_{n+1} = \{\text{Rsin } nh - \text{Rsin } (n-1)h\} - \{\text{Rsin } (n+1)h - \text{Rsin } nh\},$$

where $h = 225'$

$$= 2 \text{ Rsin } nh - \{\text{Rsin } (n+1)h + \text{Rsin } (n-1)h\}$$

$$= 2 \text{ Rsin } nh - \frac{2 \text{ Rsin } nh. \text{ Rcos } h}{R}$$

$$= 2 \text{ Rsin } nh. \frac{(R - \text{Rcos } h)}{R}$$

$$= \text{Rsin } nh. \frac{(\delta_1 - \delta_2)}{\text{Rsin } h}$$

$$= (R_n/R_1)(\delta_1 - \delta_2),$$

because

$$\delta_1 - \delta_2 = 2 \text{ Rsin } h. \frac{(R - \text{Rcos } h)}{R}.$$

The geometrical rationale as given by the commentator Nīlakaṇṭha (*c.* A.D. 1500) is as follows :

Let AOB be a quadrant of a circle, OA being horizontal and OB vertical. Let the arc AQ be equal to nh, where $h = 225'$; and let the arcs PQ and QR be each equal to h. Let L and M be the middle points of the arcs PQ and QR, so that the arc LM is also equal to h.

Let LU, QV, MW and RX be the perpendiculars on OA ; PE, LF and QD perpendiculars on QV, MW and RX, respectively. Also let PQ, LM and QR be joined by straight lines.

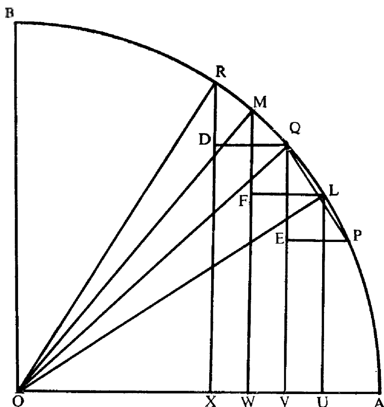


Fig. 5

Now, the triangles QEP and OUL are similar. Therefore

$$QE=(PQ/OL).\ OU.$$

Similarly,

$$RD=(QR/OM).\ OW=(PQ/OL).\ OW.$$

Therefore, by subtraction,

$$QE-RD=(PQ/OL)(OU-OW)=(PQ/OL).\ WU. \qquad (4)$$

Again, since the triangles MFL and OVQ are similar,

$$FL=(QV/OQ).\ LM,$$

$$\text{or}\quad WU=(PQ/OL).\ QV. \qquad (5)$$

From (4) and (5),

$$QE-RD=(PQ/OL)^2.\ QV.$$

In other words,

$$\delta_n \quad \delta_{n+1}=\{(2\ \text{Rsin}\ h/2)/R\}^2.\ R_n \qquad (6)$$

In particular,

$$\delta_1-\delta_2=\{(2\ \text{Rsin}\ h/2)/R\}^2.\ R_1 \qquad (7)$$

From (6) and (7),

$$\delta_n-\delta_{n+1}=(R_n/R_1)(\delta_1-\delta_2).$$

CONSTRUCTION OF CIRCLE, ETC., AND TESTING OF LEVEL AND VERTICALITY

वृत्तं भ्रमेण साध्यं त्रिभुजं च चतुर्भुजं च कर्णाभ्याम् ।
साध्या जलेन समभूरधऊर्ध्वं लम्बकेनैव ॥ १३ ॥

13. **A circle should be constructed by means of a pair of compasses ; a triangle and a quadrilateral by means of the two hypotenuses (*karṇa*). The level of ground should be tested by means of water; and verticality by means of a plumb.**[1]

The two hypotenuses (*karṇas*) in the case of a triangle are the two lateral sides above the base ; in the case of a rectangle, the two diagonals ; and in the case of a trapezium, the two lateral sides.[2] The reference is to the usual methods of constructing a triangle when the three sides (*i.e.*, the base and the two lateral sides) are given ; a parallelogram, when one side and two diagonals called the hypotenuses are given ; and a trapezium, when the base, height and the two lateral sides (called hypotenuses) are given.

As regards testing the level of the ground, Bhāskara I observes :

> "When there is no wind, place a jar (full) of water upon a tripod on the ground which has been made plane by means of eye or thread, and bore a (fine) hole (at the bottom of the jar) so that water may have continuous flow. Where the water falling on the ground spreads in a circle, there the ground is in perfect level ; where the water accumulates after departing from the circle of water, there it is low ; and where the water does not reach, there it is high."

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1. The same rule occurs in *BrSpSi*, xxii. 7 ; *ŚiDVr*, II, viii. 2 (c-d). Also see *SūSi*, iii. 1 ; *SiŚi*, I, iii. 8.

2. See examples set by Bhāskara I and Sūryadeva in their commentaries on *Ā*, ii. 6 (*a-b*).

RADIUS OF THE SHADOW-SPHERE

शङ्कोः प्रमाणवर्गं छायावर्गेण संयुतं कृत्वा ।
यत्तस्य वर्गमूलं विष्कम्भार्धं स्ववृत्तस्य ॥ १४ ॥

14. Add the square of the height of the gnomon to the square of its shadow. The square root of that sum is the semi-diameter of the circle of shadow.[1]

"The semi-diameter of the circle of shadow is taken here", says the commentator Bhāskara I, "in order to accomplish the rule of three, *viz.* 'If these are the values of the gnomon and the shadow corresponding to the radius of the circle of shadow, what will correspond to the radius of the celestial sphere ?' Thus are obtained the Rsines of the Sun's altitude and zenith distance. At an equinox, these are called the Rsines of colatitude and latitude (respectively)." *Cf. LBh,* iii. 2-3.

As regards the shape of a gnomon, Bhāskara I informs us that the Hindu astronomers differed from one another. Some took a gnomon with one third at the bottom of the shape of a right prism on a square base, one third in the middle of the shape of a cylinder, and one third at the top of the shape of a cone. Others took a gnomon of the shape of a right prism on a square base. The followers of Āryabhaṭa I, writes Bhāskara I, preferred a cylindrical gnomon, made of excellent timber, free from holes, knots and scars, with large diameter and height. In order to get a prominent tip of the shadow, a cylindrical needle (of height greater than the radius of the gnomon) made of timber or iron was fixed vertically at the top of the gnomon in the middle. Such a gnomon being large and massive was unaffected by the wind ; being cylindrical, it was easy to manufacture ; being surmounted by a needle of small diameter, the tip of the shadow was easily perceived.

A gnomon was generally divided into 12 equal parts called *aṅgulas,* but, according to Bhāskara I, there was no such hard and fast rule. A gnomon could be of any length with any number of divisions.

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1. *Cf. KK,* I, iii. 10 ; *MBh,* iii. 4.

GNOMONIC SHADOW DUE TO A LAMP-POST

शङ्कुगुणं शङ्कुभुजाविवरं शङ्कुभुजयोर्विशेषहृतम् ।
यल्लब्धं सा छाया ज्ञेया शङ्कोः स्वमूलाद्धि ॥ १५ ॥

15. Multiply the distance between the gnomon and the lamp-post (the latter being regarded as base) by the height of the gnomon and divide (the product) by the difference between (the heights of) the lamp-post (base) and the gnomon. The quotient (thus obtained) should be known as the length of the shadow measured from the foot of the gnomon.[1]

In Fig. 6, let AB be the lamp-post, CD the gnomon, and E the point where AC and BD produced meet. Then DE is the shadow cast by the gnomon due to light from the lamp at A.

Let FC be parallel to BD. Then comparing the similar triangles CDE and AFC, we have

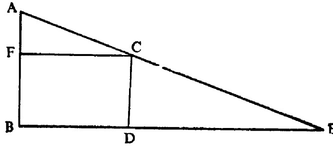


Fig. 6

$$DE = \frac{FC \times CD}{AF}$$

$$= \frac{BD \times CD}{AB - CD}.$$

Hence the rule.

TIP OF THE GNOMONIC SHADOW FROM THE LAMP-POST AND HEIGHT OF THE LATTER

छायागुणितं छायाग्रविवरमूनेन भाजितं[2] कोटी[3] ।
शङ्कुगुणा कोटी सा[4] छायाभक्ता भुजा भवति ॥ १६ ॥

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1. This rule occurs also in *BrSpSi*, xii. 53 ; *GSS*, ix. 40½ ; *SiŚe*, xiii. 54 ; *L* (*Ānandāśrama*), Rule 234, p. 243 ; *GK*, *II*, p. 208, Rule 14 (*a-b*).

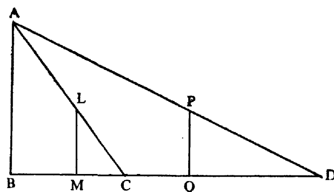
2. Bh. भाजितं ; others भाजिता   3. A-G. Gh. कोटिः

4. B. Tr. सा कोटी

16. **(When there are two gnomons of equal height in the same direction from the lamp-post), multiply the distance between the tips of the shadows (of the two gnomons) by the (larger or shorter) shadow and divide by the larger shadow diminished by the shorter one : the result is the upright (*i.e.*, the distance of the tip of the larger or shorter shadow from the foot of the lamp-post). The upright multiplied by the height of the gnomon and divided by the (larger or shorter) shadow gives the base (*i.e.*, the height of the lamp-post).**[1]

In Fig. 7, AB is the lamp-post (base), BC or BD is the upright, LM and PQ are the gnomons of equal height.

Fig. 7



We have

$$\frac{AB}{PQ}=\frac{BD}{QD} \quad \text{(i)} \qquad \frac{AB}{LM}=\frac{BC}{MC} \quad \text{(ii)}$$

Since PQ=LM, therefore

$$\frac{BD}{QD}=\frac{BC}{MC}=\frac{CD}{QD-MC} \quad \text{(iii)}$$

Hence from (iii), (i) and (ii), we have

$$BD=\frac{CD\times QD}{QD-MC} \quad (1)$$

$$BC=\frac{CD\times MC}{QD-MC} \quad (2)$$

and $$AB=\frac{BD\times PQ}{QD}=\frac{BC\times LM}{MC} \quad (3)$$

---

1. This rule reappears in *BrSpSi*, xii. 54 ; *L* (ASS), Rule 239, pp. 246-47 ; *GK*, vol. 2, p. 210, Rule 16.

THEOREMS ON SQUARE OF HYPOTENUSE AND ON SQUARE OF HALF-CHORD

यश्चैव भुजावर्गः कोटीवर्गश्च कर्णवर्गः सः[1] ।
वृत्ते शरसंवर्गोऽर्धज्यावर्गः य खलु धनुषोः ॥ १७ ॥

17 **(In a right-angled triangle) the square of the base plus the square of the upright is the square of the hypotenuse.**

In a circle (when a chord divides it into two arcs), the product of the arrows of the two arcs is certainly equal to the square of half the chord

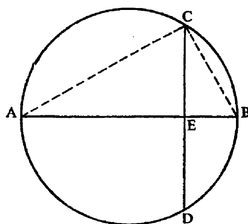
Of the two theorems stated above, the first one is "the theorem of the Square of the Hypotenuse", as Hankel has called it. This theorem has been known in India since very early times. Baudhāyana (*c.* 800 B.C.), the author of the *Baudhāyana-śulba-sūtra,* has enunciated it thus :

> "The diagonal of a rectangle produces both (areas) which its length and breadth produce separately."[2]

This theorem is now universally associated with the name of the Greek Pythagoras (*c.* 540 B.C.), though "no really trustworthy evidence exists that it was actually discovered by him." It were certainly the Hindus who enunciated the property of the right-angled triangle in its most general form. No other ancient nation is known to have made any attempt in this direction.

The second theorem states that if, in a circle, a chord CD and a diameter AB intersect each other at right-angles at E, then

Fig. 8



$AE \times EB = CE^2$.

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1. Pr. भुजावर्गयुतः कोटीवर्गः कर्णस्स एव
2. *Baudhāyana-śulba-sūtra*, i. 48.
3. Heath, *Greek Mathematics*, I, p. 144 f,

This result easily follows from comparison of the similar triangles CAE and CEB.

This second theorem occurs earlier in the works of Umāsvāti[1] (1st century A.D.). It has been mentioned by Jinabhadra Gaṇi[2] (A.D. 609) and Brahmagupta (A.D. 628) also.[3]

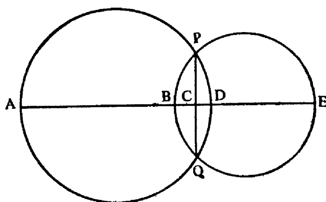
ARROWS OF INTERCEPTED ARCS OF INTERSECTING CIRCLES

ग्रासोने द्वे वृत्ते ग्रासगुणे भाजयेत् पृथक्त्वेन[4] ।
ग्रासोनयोगलब्धौ[5] सम्पातशरौ परस्परतः ॥ १८ ॥

18. **(When one circle intersects another circle) multiply the diameters of the two circles each diminished by the erosion, by the erosion and divide (each result) by the sum of the diameters of the two circles after each has been diminished by the erosion : then are obtained the arrows of the arcs (of the two circles) intercepted in each other.**[6]

Let two circles intersect at P and Q and let ABCDE be the line passing through the centres of the two circles. Then BD is the erosion (*grāsa*), and BC, CD are the arrows of the intercepted arcs.

Fig. 9



The rule states that

$$BC = \frac{(AD-BD).\ BD}{(AD-BD)+(BE-BD)} \quad \ldots \quad (1)$$

1. See *Tattvārthādhigama-sūtra*, iii. 11 (comm.) and *Jambū-dvīpa-samāsa*, ch. iv.
2. See *Bṛhat-kṣetra-samāsa*, i. 36.
3. See *BrSpSi*, xii. 41.
4. Gh. पृथक् तेन
5. A. D. Pa. योगभक्ते ; E. योगभक्तौ
6. This rule occurs also in *BrSpSi*, xii 42 ; *GSS*, vii. 231½ ; *GK*. vol. 2, p. 76 (Rule 68).

$$\text{and} \quad CD = \frac{(BE-BD).\ BD}{(AD-BD)+(BE-BD)} \quad \ldots \quad (2)$$

These formulae may be derived as follows :

Since $AC \times CD = BC \times CE$, therefore

$$(AD-BD+BC)(BD-BC) = BC(BE-BC) \quad (3)$$

$$(AD-CD)CD = (BD-CD)(BE-BD+CD) \quad (4)$$

Solving (3) for BC, we get (1), and solving (4) for CD, we get (2).

SUM (OR PARTIAL SUM) OF A SERIES IN A.P.

इष्टं व्येकं दलितं सपूर्वमुत्तरगुणं समुखमध्यम्[1] ।
इष्टगुणितमिष्टधनं त्वथवाद्यन्तं[2] पदार्धहतम् ॥ १९ ॥

19. **Diminish the given number of terms by one, then divide by two, then increase by the number of the preceding terms (if any), then multiply by the common difference, and then increase by the first term of the (whole) series : the result is the arithmetic mean (of the given number of terms). This multiplied by the given number of terms is the sum of the given terms. Alternatively, multiply the sum of the first and last terms (of the series or partial series which is to be summed up) by half the number of terms.**[3]

Let an arithmetic series be

$$a+(a+d)+(a+2d)+\ldots\ldots$$

Then the rule says that

(1) the arithmetic mean of the n terms

$$(a+pd)+(a+\overline{p+1}\,d)+\ \ldots\ +\{a+(p+n-1)\,d\}$$
$$=a+\left(\frac{n-1}{2}+p\right)d.$$

---

1. Bh. समुखं मध्यम्
2. C. धनमथवाद्यन्तं
3. *Cf. BrSpSi*, xii. 17 ; *GSS*, vi. 290 ; *PG*, Rule 85 ; *MSi*, xv. 47 ; *SiŚe*, xiii. 20 ; *L (ASS)*, Rule 121, p. 114.

(2) the sum of the n terms

$$(a+pd)+(a+\overline{p+1}d)+\ \dots\ +\{a+(p+n-1)\ d\}$$
$$= n\left\{a+\left(\frac{n-1}{2}+p\right)d\right\}.$$

In particular (when $p=0$)

(3) the arithmetic mean of the series

$$a+(a+d)+\dots+\{a+(n-1)d\}$$
$$=a+\frac{n-1}{2}d$$

(4) the sum of the series

$$a+(a+d)+\dots+\{a+(n-1)\ d\}$$
$$= n\left\{a+\frac{n-1}{2}d\right\}$$

Alternatively, the sum of n terms of an arithmetic series with A as the first term and L as the last term

$$= \frac{n}{2}(\mathrm{A}+\mathrm{L}),$$

where $\frac{1}{2}(\mathrm{A}+\mathrm{L})$ is the arithmetic mean of the terms.

The commentator Bhāskara I says :

"Several formulae are severally set out here. They are obtained by suitable combination of the text as follows :

Formula 1. *"Iṣṭaṁ vyekaṁ dalitaṁ uttaraguṇaṁ samukham" iti madhyadhanānayanārthaṁ sūtram.*

i.e., $a+\frac{n-1}{2}d$ is the formula for the arithmetic mean of n terms.

Formula 2. *"Madhyaṁ iṣṭaguṇitam" iti iṣṭadhanam.*

i.e., $\left\{a+\frac{n-1}{2}d\right\}n$ gives the sum of n terms.

Formula 3. *"Iṣṭaṁ vyekaṁ sapūrvam uttaraguṇaṁ samukham" ityantyopāntyādidhanānayanārthaṁ sūtram.*

i.e., $a+\{(1-1)+(n-1)\}d$ is the formula for the nth term.

Formula 4. *"Iṣṭaṁ vyekaṁ dalitaṁ sapūrvam uttaraguṇaṁ samukham iṣṭaguṇitam" ityavāntarayatheṣṭapadasaṁkhyānayanārthaṁ sūtram.*

i.e., $n\left\{a + \left(\frac{n-1}{2} + p\right) d\right\}$ is the formula for the sum of n terms beginning with the $(p+1)$th term.

Formula 5. *"Ādyantapadārdhahatam" iti iṣṭadhanam.*

i.e., $\frac{1}{2}n$ (A+L) is the sum of n terms, A and L being the first and last terms.

Series in A.P. are found to occur in the *Taittirīya Saṁhitā* (vii. 2. 12-17; iv. 3. 10), the *Vājasaneya Saṁhitā* (xvii. 24. 25), the *Pañcaviṁśa Brāhmaṇa* (xviii. 3) and other Vedic works. In the *Bṛhaddevatā*[1] (500-400 B.C.), we have the result

$$2+3+4+ \dots \dots +1000=500499.$$

Formal rules for finding the sum etc. of a series in A.P. occur in the Bakshali Manuscript (*c*. 200 A.D.) and other Indian works on mathematics written subsequently.

NUMBER OF TERMS OF A SERIES IN A.P.

गच्छोऽष्टोत्तरगुणिताद् द्विगुणाद्युत्तरविशेषवर्गयुतात् ।
मूलं द्विगुणाद्यूनं स्वोत्तरभजितं सरूपार्धम् ॥ २० ॥

20. The number of terms (is obtained as follows): Multiply (the sum of the series) by eight and by the common difference, increase that by the square of the difference between twice the first term and the common difference, and then take the square root; then subtract twice the first term, then divide by the common difference, then add one (to the quotient), and then divide by two.[2]

Let S be the sum of the series

$$a+(a+d)+(a+2d)+(a+3d)+\dots\dots \text{to } n \text{ terms.}$$

1. The *Bṛhaddevatā* has been edited in original Sanskrit with English translation by Macdonell, Harvard, 1904.

2. *Cf. BrSpSi*, xii. 18; *GSS*, vi. 294; *PG*, Rule 87; *MSi*, xv. 50; *SiŚe*, xiii. 24; *L* (ASS), Rule 128, p. 118.

Then

$$n=\frac{1}{2}\left[\frac{\sqrt{8dS+(2a-d)^2}-2a}{d}+1\right].$$

SUM OF THE SERIES 1+(1+2)+(1+2+3)+... ... TO N TERMS

एकोत्तराद्युपचितेर्गच्छाद्येकोत्तरत्रिसंवर्गः ।
षड्भक्तः स चितिघनः सैकपदघनो विमूलो वा ॥ २१ ॥

21. **Of the series (*upaciti*) which has one for the first term and one for the common difference, take three terms in continuation, of which the first is equal to the given number of terms, and find their continued product. That (product), or the number of terms plus one subtracted from the cube of that, divided by 6, gives the *citighana*.**[1]

The term *upaciti* or *citi* is used in the sense of a series in gêneral. The series $1+2+3+\dots\dots+n$, which has one for the first term and one for the common difference is called *ekottarādi-upaciti.* The sum of this series is generally called *saṅkalita*. Bhāskara I calls it *saṅkalanā.*

The term *citighana* is used in the sense of the sum of the series

$$1+(1+2)+(1+2+3)+\dots\dots \tag{1}$$

to any number of terms. This sum is generally called *saṅkalita-saṅkalita.* Bhāskara I has called it *saṅkalanā-saṅkalanā.*

The above rule gives the sum to n terms of the series (1) in two forms :

$$\text{(i)}\quad \frac{n(n+1)(n+2)}{6} \quad\text{and}\quad \text{(ii)}\quad \frac{(n+1)^3-(n+1)}{6}.$$

The term *citighana* literally means 'the solid contents of a pile (of balls) in the shape of a pyramid on a triangular base'. The pyramid is so constructed that there is 1 ball in the topmost layer, $1+2$ balls in the next lower layer, $1+2+3$ balls in the further next lower layer, and so on. In the nth layer, which forms the base, there are

$$1+2+3+\dots+n \text{ balls.}$$

---

1. *Cf. BrSpSi*, xii. 19 ; *PG*, Rule 103 (c-d) ; *SiŚe*, xiii, 21 ; *L* (ASS), Rule 118, p. 112.

The number of balls in the solid pyramid,

$$\text{i.e., } citighana = S_1 + S_2 + \dots + S_r + \dots + S_n,$$

$$\text{where} \quad S_r = 1 + 2 + 3 + \dots + r.$$

The base of the pyramid is called *upaciti*, so

$$upaciti = 1 + 2 + 3 + \dots + n.$$

Bhāskara I illustrates the rule by the following example :

Example. There are (three pyramidal) piles (of balls) having respectively 5, 8 and 14 layers which are triangular. Tell me the number of units (*i.e.*, balls) (in each of them).

The above *citighana* is a series of figurate numbers. The Hindus are known to have obtained the formula for the sum of the series of natural numbers as early as the fifth century B.C. It cannot be said with any certainty whether the Hindus in those times used the representation of the sum by triangles or not. The subject of piles of shots and other things has been given great importance in the Hindu works, of which all contain a section dealing with *citi* ('piles'). It will not be a matter of surprise if the geometrical respresentation of figurate numbers is traced to Hindu sources.

SUM OF THE SERIES ΣN^2 AND ΣN^3

सैक-सगच्छ-पदानां क्रमात् त्रिसंवर्गितस्य षष्ठोंऽशः ।
वर्गचितिघनः स भवेत्, चितिवर्गो घनचितिघनश्च ॥ २२ ॥

22. **The continued product of the three quantities, *viz.*, the number of terms plus one, the same increased by the number of terms, and the number of terms, when divided by 6 gives the sum of the series of squares of natural numbers (*vargacitighana*). The square of the sum of the series of natural numbers (*citi*) gives the sum of the series of cubes of natural numbers (*ghanacitighana*).**[1]

1. *Cf. BrSpSi*, xii. 20 ; *PG*, Rule 102-3 (*a-b*), *SiŚe*, xiii. 22 ; *L* (ASS), Rule 119, p. 113.

The term *vargacitighana* is used in the sense of the sum of the series

$$1^2+2^2+3^2+ \dots + n^2,$$

i.e., the sum of the series of squares of natural numbers ; and the term *ghanacitighana* is used in the sense of the sum of the series

$$1^3+2^3+3^3+ \dots + n^3,$$

i.e., the sum of the series of cubes of natural numbers. Bhāskara I has called these sums by the terms *vargasaṅkalanā* and *ghanasaṅkalanā*, respectively. Other mathematicians have called them *vargasaṅkalita* and *ghanasaṅkalita*, respectively. According to the above rule

$$1^2+2^2+3^2+\dots+n^2=\frac{n\,(n+1)(2n+1)}{6}, \qquad \text{and}$$

$$1^3+2^3+3^3+ \dots +n^3=(1+2+3+ \dots +n)^2$$

$$=\left\{\frac{n\,(n+1)}{2}\right\}^2$$

The term *vargacitighana* literally means 'the solid contents of a pile (of balls) in the shape of a pyramid on a square base'. It is so constructed that there is 1 ball in the topmost layer, 2^2 balls in the next lower layer, 3^2 balls in the further next lower layer, and so on. In the nth layer from the top, which forms the base of the pile, there are n^2 balls.

The term *ghanacitighana* similarly means 'the solid contents of a pile (of cuboidal bricks) in the shape of a pyramid having cuboidal layers'. It is so constructed that there is 1 brick in the topmost layer, 2^3 bricks in the next lower layer, 3^3 bricks in the further next lower layer, and so on. In the nth layer (from the top), which forms the base of the pile, there are n^3 bricks, n bricks in each edge of the cuboidal base.

Bhāskara I illustrates Āryabhaṭa's rule stated in the text by the following examples :

Example 1. There are (three pyramidal) piles on square bases having 7, 8 and 17 layers which are also squares. Say the number of units therein (*i.e.*, the number of balls or bricks, of unit size used in each of them).

Example 2. There are (three pyramidal) piles having 5, 4 and 9 cuboidal layers. They are constructed of cuboidal bricks (of unit dimensions) with one brick in the topmost layer. (Find the number of bricks used in each of them).

PRODUCT OF FACTORS FROM THEIR SUM AND SQUARES

सम्पर्कस्य हि वर्गाद् विशोधयेदेव वर्गसम्पर्कम् ।
यत्तस्य भवत्यर्धं विद्याद् गुणकारसंवर्गम् ॥ २३ ॥

23. From the square of the sum of the two factors subtract the sum of their squares. One-half of that (difference) should be known as the product of the two factors.

That is,

$$A \times B = \frac{(A+B)^2 - (A^2+B^2)}{2}.$$

QUANTITIES FROM THEIR DIFFERENCE AND PRODUCT

द्विकृतिगुणात् संवर्गाद् द्व्यन्तरवर्गेण संयुतान्मूलम् ।
अन्तरयुक्तं हीनं तद् गुणकारद्वयं दलितम् ॥ २४ ॥

24. Multiply the product by four, then add the square of the difference of the two (quantities), and then take the square root. (Set down this square root in two places). (In one place) increase it by the difference (of the two quantities), and (in the other place) decrease it by the same. The results thus obtained, when divided by two, give the two factors (of the given product).[1]

That is, if

$$x - y = a$$
$$xy = b,$$

then

$$x = \frac{\sqrt{4b+a^2} + a}{2}$$
$$y = \frac{\sqrt{4b+a^2} - a}{2}.$$

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1. *Cf. BrSpSi*, xviii. 99.

INTEREST ON PRINCIPAL

मूलफलं सफलं कालमूलगुणमर्धमूलकृतियुक्तम् ।
तन्मूलं मूलार्धोनं कालहृतं स्वमूल[1]फलम्[2] ॥ २५ ॥

25. **Multiply the interest on the principal plus the interest on that interest by the time and by the principal; (then) add the square of half the principal; (then) take the square root; (then) subtract half the principal; and (then) divide by the time: the result is the interest on the principal.[3]**

The problem envisaged is: A principal P is lent out at a certain rate of interest per month. At the expiry of one month, the interest 'I' which accrues on P in one month is given on loan at the same rate of interest for T months. After T months 'I' amounts to A. The problem is to find 'I' when A is given.

The solution to this problem is

$$I = \frac{\sqrt{PTA+(P/2)^2}-(P/2)}{T}.$$

as stated in the above rule.

RULE OF THREE

त्रैराशिकफलराशिं तमथेच्छाराशिना हतं कृत्वा ।
लब्धं प्रमाणभजितं तस्मादिच्छाफलमिदं स्यात् ॥ २६ ॥

26. **In the rule of three, multiply the 'fruit' (*phala*) by the 'requisition' (*icchā*) and divide the resulting product by the 'argument' (*pramāṇa*). Then is obtained the 'fruit corresponding to the requisition' (*icchāphala*).[4]**

---

1. B. स्वमूल crossed out and समूल substituted.

2. Bh. and So. read तन्मूलं मूलार्धोनं कालहृतं स्वमूलफलम् ; others read मूलं मूलार्धोनं कालहृतं स्यात् स्वमूलफलम् ।

3. Brahmagupta gives a more general rule. See *BrSpSi*, xii. 15.

4. Similar rules occur in *BrSpSi*, xii. 10 ; *GSS*, v. 2 (i) ; *PG*, Rule 43 ; *MSi*, xv 24-25 (*a-b*); *GT*, p. 68, vs. 86 ; *SiŚe*, xiii. 14 ; *L* (ASS), p. 71, vs. 73 ; *GK*, I, p. 47, vs. 60.

Example 1. If A books cost P rupees, what will R books cost ?

Here A is the 'argument', P the 'fruit' and R the 'requisition'. So the required answer is

$$\frac{P \times R}{A} \text{ rupees.}$$

Example 2. If the interest on Rs. 100 for 2 months is Rs. 5, find the interest on Rs. 25 invested for 8 months.

Here we have two arguments, *viz,* Rs. 100 and 2 months ; and two requisitions *viz.,* Rs. 25 and 8 months. The fruit is Rs. 5. So the required answer is

$$\frac{25 \times 8 \times 5}{100 \times 2} \text{ or 5 rupees.}$$

SIMPLIFICATION OF THE QUOTIENTS OF FRACTIONS

छेदाः परस्परहता भवन्ति गुणकारभागहाराणाम् ।

27. (a-b) The numerators and denominators of the multipliers and divisors should be multiplied by one another.

For example,

$$\text{(i)} \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

$$\text{(ii)} \quad \frac{\frac{a}{b} \times \frac{c}{d}}{\frac{e}{f} \times \frac{g}{h}} = \frac{\frac{ac}{bd}}{\frac{eg}{fh}} = \frac{(ac)\,(fh)}{(bd)\,(eg)}$$

This rule is a sequel to the previous rule of three, and relates to the case when the argument, fruit and requisition are each fractional. What is meant int his rule is that when the fractional fruit and the fractional requisition have been multiplied and a fractional product is obtained, then the product should be treated as the multiplier and the argument as the divisor. The numerator of the multiplier should then be multiplied by the denominator of the divisor and the denominator of the

multiplier by the numerator of the divisor. The commentator Sūryadeva explains : "Here by the word *guṇakāra* (multiplier) are meant the fruit and the requisition, because, being the multiplicand and the multiplier, both of them are mutually multipliers. By the word *bhāgahāra* (divisor) is meant the argument. The (product of the) denominators of the fruit and the requisition should be multiplied by the (numerater of the) argument, and the product of (the numerators of) the fruit and the requisition should be multiplied by the denominator of the argument."

REDUCTION OF TWO FRACTIONS TO A COMMON DENOMINATOR

छेदगुणं सच्छेदं परस्परं तत् सवर्णत्वम् ॥ २७ ॥

27. (c-d) Multiply the numerator as also the denominator of each fraction by the denominator of the other fraction ; then the (given) fractions are reduced to a common denominator.[1]

That is,

$$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd},$$

$$\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}.$$

Example. Add $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$.

Reducing $\frac{1}{2}$ and $\frac{1}{6}$ to a common denominator and adding, we get

$$\tfrac{1}{2}+\tfrac{1}{6}=\frac{6}{12}+\frac{2}{12}=\frac{6+2}{12}=\tfrac{2}{3}$$

Now adding $\frac{2}{3}$ and $\frac{1}{3}$, we get

$$\tfrac{2}{3}+\tfrac{1}{3}=\frac{2+1}{3}=1.$$

1. Similar rules occur in *BrSpSi*, xii. 2 (a-b) ; *PG*, Rule 36 ; *MSi*, xv. 13 (c-d) ; *GT*, p. 30, line 16 ; *SiŚe*, xiii. 11 (a-b) ; *L* (ASS), p. 28, lines 9 ; *GK*, I, p. 9, vs. 26 (a-b). Also see *GSS*, iii. 55 (a-b).

METHOD OF INVERSION

गुणकारा भागहरा भागहरास्ते[1] भवन्ति गुणकाराः ।
यः क्षेपः सोऽपचयोऽपचयः क्षेपश्च[2] विपरीते[3] ॥ २८ ॥

28. **In the method of inversion multipliers become divisors and divisors become multipliers, additive becomes subtractive and subtractive becomes additive.**[4]

Example. A number is multiplied by 2; then increased by 1; then divided by 5; then multiplied by 3; then diminished by 2; and then divided by 7; the result (thus obtained) is 1. Say what is the initial number.

Starting from the last number 1, in the reverse order, inverting the operations, the result is

$1 \times 7, + 2, \div 3, \times 5, -1, \div 2,$ *i. e.* 7.

UNKNOWN QUANTITIES FROM SUMS OF ALL BUT ONE

राश्यूनं राश्यूनं गच्छधनं पिण्डितं पृथक्त्वेन ।
व्येकेन पदेन हृतं सर्वधनं तद् भवत्येव[5] ॥ २९ ॥

29. **The sums of all (combinations of) the (unknown) quantities except one (which are given) separately should be added together; and the sum should be written down separately and divided by the number of (unknown) quantities less one: the quotient thus obtained is certainly the total of all the (unknown) quantities. (This total severally diminished by the given sums gives the various unknown quantities).**[6]

---

1. Gh. Go. Nī. Pa. So. भागहरा ये
2. Ra. क्षेपः स
3. C. क्षेपश्च भवति विपरीते
4. Similar rules occur in *BrSpSi*, xviii. 14; *GSS*, vi. 286; *PG*, Rule 78; *MSi*, xv. 23; *GT*, p. 65, vs. 83; *SiŚe*, xiii. 13; *L* (ASS), p. 42, vs. 48; *GK*, I, p. 46, lines 13-16.
5. Bh. भवत्येवम्
6. *Cf. GSS*, vi. 159; *GK*, I, p. 85, Rule 28.

That is if

$$(x_1+x_2+\ \dots\ +x_n)-x_1=a_1$$
$$(x_1+x_2+\ \dots\ +x_n)-x_2=a_2$$
$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$
$$(x_1+x_2+\ \dots\ +x_n)-x_n=a_n$$

then

$$x_1+x_2+\ \dots\ +x_n=\frac{a_1+a_2+\dots+a_n}{n-1},$$

so that

$$x_1=\frac{a_1+a_2+\ \dots\ +a_n}{n-1}-a_1$$
$$x_2=\frac{a_1+a_2+\ \dots\ +a_n}{n-1}-a_2,$$
$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$
$$x_n=\frac{a_1+a_2+\ \dots\ +a_n}{n-1}-a_n$$

$x_1, x_2, \dots, x_n$ being the unknown quantities and $a_1, a_2, \dots, a_n$ the given sums.

UNKNOWN QUANTITIES FROM EQUAL SUMS

गुलिकान्तरेण विभजेद् द्वयोः पुरुषयोस्तु रूपकविशेषम् ।
लब्धं गुलिकामूल्यं यद्यर्थकृतं भवति तुल्यम्[1] ॥ ३० ॥

30. Divide the difference between the *rūpakas* with the two persons by the difference between their *gulikās*. The quotient is the value of one *gulikā*, if the possessions of the two persons are of equal value.[2]

---

1. A. B. E-G. भवति तत्तुल्यम्

2. The verse may also be translated as: "The difference of the known amounts (*rūpaka*) relating to the two persons should be divided by the difference of the coefficients of the unknowns (*gulikā*): The quotient will be the value of the unknown; if their possessions be equal." See B. Datta and A.N. Singh, *History of Hindu Mathematics*, part II, p. 40.

For similar rules see *BrSpSi*, xviii. 43; *SiŚe*, xiv. 15; *BB* (ASS), p. 113, Rule 89; *NBī*, II, Rule 5.

Two persons are equally rich. Of them, one possesses *a gulikās* and *b rūpakas* (coins), and the other possesses *c gulikās* and *d rūpakas*. The rule tells how to find the value of one *gulikā* in terms of *rūpakas*.

Algebraically, if

$$ax+b=cx+d,$$

then

$$x=\frac{d-b}{a-c}.$$

The term *gulikā* stands for 'a thing of unknown value'. "By the term *gulikā*," writes Bhāskara I (629 A.D.), "is expressed a thing of unknown value." *Gulikā* and *yāvattāvat* (commonly used in Hindu algebra for an unknown) are used as synonyms. Bhāskara I writes : "These very *gulikās* of unknown value are called *yāvattāvat*."

The term *rūpaka* means a coin. "The *rūpaka*", writes Bhāskara I, "is (a coin such as) *dīnāra* etc."

MEETING OF TWO MOVING BODIES

भक्ते विलोमविवरे गतियोगेनानुलोमविवरे द्वौ[1] ।
गत्यन्तरेण लब्धौ[2] द्वियोगकालावतीतैष्यौ ॥ ३१ ॥

31. Divide the distance between the two bodies moving in the opposite directions by the sum of their speeds, and the distance between the two bodies moving in the same direction by the difference of their speeds ; the two quotients will give the time elapsed since the two bodies met or to elapse before they will meet.[3]

---

1. B. F. N. विवरौ द्वौ ; Go. Pa. So. विवरे द्वे

2. Nī. भक्तौ

3. Problems on meeting of travellers occur in *Bakhshali Manuscript* and later works. *BM*, III, A_{13}, 3 recto, Rule 14 ; B_2, 9 verso ; B_4, 4 recto. ; *GSS*, vi. 326½ ; *PG*, Rule 65, Exs. 81-82.

The following cases may arise :

Case 1. When the two bodies are moving in the opposite directions.

If the bodies are facing each other, *i.e.*, if they have not already met, the distance between them when divided by the sum of their velocities will give the time to elapse before they meet.

If the bodies have already met and moving away from each other, the distance between them when divided by the sum of their velocities will give the time elapsed since they met each other.

Case 2. When the two bodies are moving in the same direction.

If the fast-moving body is behind, *i.e.*, if they have not already met, the distance between them when divided by the difference of their velocities will give the time to elapse before they meet.

If the slow-moving body is behind, *i.e.*, if they have already met, the distance between them when divided by the difference of their velocities will give the time elapsed since they met each other.

PULVERISER

The rule stated in vss. 32-33 below is meant for solving a residual pulveriser (*sāgra-kuṭṭākāra*), *i.e.*, a problem of the following type :

> A number leaves 1 as the remainder when divided by 5, and 2 (as the remainder) when divided by 7. Calculate what that number is.

RESIDUAL PULVERISER

अधिकाग्रभागहारं छिन्द्यादूनाग्रभागहारेण ।
शेषपरस्परभक्तं मतिगुणमग्रान्तरे क्षिप्तम् ॥ ३२ ॥

अधउपरिगुणितमन्त्ययुगूनाग्रच्छेदभाजिते शेषम् ।
अधिकाग्रच्छेदगुणं द्विच्छेदाग्रमधिकाग्रयुतम् ॥ ३३ ॥[1]

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1. A. No colophon ; B. D. इति गणितपादः ; C. इति गणितपादः समाप्तः E. गणितपादम् ; F. इति गणितक्रिया समाप्तम् ।

32-33. Divide the divisor corresponding to the greater remainder by the divisor corresponding to the smaller remainder. (Discard the quotient). Divide the remainder obtained (and the divisor) by one another (until the number of quotients of the mutual division is even and the final remainder is small enough). Multiply the final remainder by an optional number and to the product obtained add the difference of the remainders (corresponding to the greater and smaller divisors; then divide this sum by the last divisor of the mutual division. The optional number is to be so chosen that this division is exact. Now place the quotients of the mutual division one below the other in a column; below them write the optional number and underneath it the quotient just obtained. Then reduce the chain of numbers which have been written down one below the other, as follows): Multiply by the last but one number (in the bottom) the number just above it and then add the number just below it (and then discard the lower number). (Repeat this process until there are only two numbers in the chain). Divide (the upper number) by the divisor corresponding to the smaller remainder, then multiply the remainder obtained by the divisor corresponding to the greater remainder, and then add the greater remainder: the result is the *dvicchedāgra* (i.e., the number answering to the two divisors). (This is also the remainder corresponding to the divisor equal to the product of the two divisors).[1]

It may be pointed out that when the quotients of the mutual division are odd in number, the difference of the greater and smaller remainders is subtracted from the product of the last remainder of the mutual division and the optional number.

To illustrate the above rule, we solve the following example.

Example. Find the number which yields 5 as the remainder when divided by 8, 4 as the remainder when divided by 9, and 1 as the remainder when divided by 7.

---

1. The same rule occurs in *BrSpSi*, xviii. 3-5.

	(i)	(ii)	(iii)
Remainder	5	4	1
Divisor	8	9	7

To begin with, we apply the process of the pulveriser on the first two pairs of remainder and divisor, *viz.*, (i) and (ii).

Dividing 8 (the divisor corresponding to the greater remainder) by 9 (the divisor corresponding to the smaller remainder), we get 8 as the remainder and 0 as the quotient. We discard the quotient 0 and divide the remainder 8 and the divisor 9 mutually until there are even number of quotients and the final remainder is small :

```
8) 9 ( 1
   8
   ‾‾
   1 ) 8 (8
       8
       ‾
       0
```

We choose 1 as the optional number and multiply the remainder 0 by it and add 1 (the difference of the greater and smaller remainders) to it. The result is 1. Diving this 1 by 1 (the final divisor of the mutual division), the quotient obtained is 1. Now, we write the quotients of the mutual division, *viz.*, 1 and 8 one below the other and below them the optional number 1 and then the quotient 1 just obtained. Thus we get

1
8
1
1

Reducing this chain, we successively get

1	1	10
8	9	9
1	1	
1		

Now dividing the upper number 10 by 9 (the divisor corresponding to the smaller remainder), we get 1 as the remainder. Multiplying this 1 by 8 (the divisor corresponding to the greater remainder), we get 8. Adding the greater remainder 5 to this 8, we get 13.

This 13 is obviously the number which divided by 8 leaves 5 as remainder and divided by 9 leaves 4 as remainder. This number is called *dvicchedāgra* because this answers to two divisors.

This 13 is also the remainder corresponding to the divisor 8×9 (*i.e.*, 72).

We now apply the process of the pulveriser on the following pairs of remainder and divisor :

	(i)	(ii)
Remainder	13	1
Divisor	72	7

Proceeding as above, we get 85. This is called *tricchedāgra* because this answers to three divisors (viz., those in the example). One can easily see that 85 leaves 5 as remainder when divided by 8, 4 as remainder when divided by 9, and 1 as remainder when divided by 7.

This is the least integral solution of the problem. The general solution is $8 \times 9 \times 7\ m+85$, *i.e.*, $504\ m+85$, $m=0, 1, 2, 3, \ldots$

The commentators Bhāskara I, Sūryadeva and others have also interpreted vss. 32-33 as a rule for solving a non-residual pulveriser (*niragra-kuṭṭākāra*), *i.e.*, a problem of the following type :

Problem : 11 is multiplied by a certain number, the product is diminished by 3, and the difference thus obtained being divided by 23 is found to be exactly divisible. Find the multiplier and the quotient.

NON-RESIDUAL PULVERISER

Verses 32-33 may be translated also as follows :

32-33. Divide the greater number (denoting the divisor) by the smaller number (denoting the dividend) (and by the remainder

obtained the smaller number and so on. Dividing the greater and the smaller numbers by the last non-zero remainder of the mutual division, reduce them to their lowest terms.[1]) Divide the resulting numbers mutually (until the number of quotients of the mutual division is even and the final remainder is small enough). Multiply the final remainder by an optional number and to the product obtained add the (given) additive (or subtract the subtractive).[2] (Divide this sum or difference by the last divisor of the mutual division. The optional number is so chosen that this division is exact. Now place the quotients of the mutual division one below the other in a column ; below them write the optional number and underneath it the quotient just obtained. Then reduce this chain of numbers as follows). Multiply by the last but one number (in the bottom) the number just above it and then add the number just below it (and then discard the lower number). (Repeat this process until there are only two numbers in the chain). Divide (the upper number by the abraded greater number and the lower number) by the abraded smaller number. (The remainders thus obtained are the required values of the unknown multiplier and quotient).

In the above translation, the word *agra* has been taken to mean 'number' and *agrāntara* to mean 'the given additive or subtractive'.

The operations mentioned in *adhikāgracchedaguṇaṁ dvicchedāgram adhikāgrayutam* are, as remarked by the commentator Sūryadeva, not needed in the case of a non-residual pulveriser. Writes he : "*Adhikāgracchedaguṇam ityādi niragrakuṭṭakāreṣu nopayujyate.*"

This rule might be illustrated as follows :

Example. Solve

$$\frac{16\,x-138}{487}=y.$$

---

1. The additive or subtractive should also be reduced by dividing it by the last non-zero remainder.

2. By the additive and subtractive here is meant the reduced additive and reduced subtractive.

Here, the divisor=487, dividend=16 and subtractive=138. Since 487 and 16 are already prime to each other, we proceed with their mutual division. The mutual division runs as follows :

```
16) 487 (30
    480
    ---
     7) 16 (2
        14
        --
         2×76—138= 14 (2
                   14
                   --
                    0
```

The chain of the quotients of the mutual division, the optional number and the final quotient is reduced as follows :

30	30	4696
2	154	154
76	76	
2		

Dividing 4696 by the divisor 487, the remainder is 313 : this is the value of x. Dividing 154 by the dividend 16, the remainder is 10 : this is the value of y.

Hence $x=313$, $y=10$.

This is the least integral solution of the problem.

The general solution is

$$x=487\lambda+313$$
$$y=16\lambda+10$$

where $\lambda=0, 1, 2, 3 \ldots$

The commentator Someśvara, instead of interpreting the text in a different way in the case of a non-residual pulveriser, interprets a non-residual pulveriser itself as a residual pulveriser.[1] Thus, he

1. This is really the method of Brahmagupta. See *BrSpSi*, xviii. 7.

interprets the non-residual pulveriser

$$\frac{ax-c}{b}=y,$$

as the residual pulveriser

$$N=by+c=ax+0,$$

in which

c is the *adhikāgra* (*i.e.*, the greater remainder),

b is the *adhikāgrabhāgahāra* (*i.e.*, divisor corresponding to the greater remainder),

0 is the *ūnāgra* (*i.e.*, the smaller remainder), and

a is the (*ūnāgrabhāgahāra*, *i.e.*, the divisor corresponding to the smaller remainder).

To illustrate this method, we solve the non-residual pulveriser of *Example* 2, *viz.*,

$$\frac{16x-138}{487}=y$$

by converting it into a residual pulveriser.

This is equivalent to the residual pulveriser

$$N=16x+0=487y+138,$$

where the greater remainder $=138$, the corresponding divisor 487, smaller remainder $=0$, and corresponding divisor $=16$. To solve this, we proceed as in *Example* 1.

The mutual division runs as follows :

```
16) 487 (30
    480
    ---
     7) 16 (2
        14
        --
         2) 7 (3
            6
            -
            1×2+138        optional number=2
            -------
                140 (70
                140
                ---
                  0
```

We discard the first quotient and write down the other quotients of the mutual division one below the other in a column, and underneath them the optional number and the final quotient ; and then reduce the chain of numbers obtained, as follows :

$$\begin{array}{ccc} 2 & 2 & 154 \\ 3 & 76 & 76 \\ 2 & 2 & \\ 70 & & \end{array}$$

Dividing 154 by 16 (the divisor corresponding to the smaller remainder), the remainder is 10 ; this 10 multiplied by 487 (the divisor corresponding to the greater remainder) gives 4870 ; this increased by 138 (the greater remainder) gives 5008. This is the value of N (the *dvicchedāgra*) The values of x and y are, therefore, the following :

$$x = \frac{5008}{16} = 313$$

$$y = \frac{5008-138}{487} = 10.$$

The rationale of Āryabhaṭa I's rule is as follows :

Method 1. Since both residual and non-residual pulverisers reduce ultimately to an equation of the form

$$ax + c = by, \tag{1}$$

it is sufficient to start with this equation.

Let the mutual division of a and b $(b > a)$ yield

$$\begin{array}{l} a)\ b\ (q_1 \\ \overline{r_1)}\ a\ (q_2 \\ \quad\ \overline{r_2} \end{array}$$

The mutual division may be continued to any number of even quotients. For convenience, we have stopped after obtaining the second quotient.

The mutual division shows that the application of pulverisation to equation (1) gives rise to the equation

$$ax+c=r_1y \tag{2}$$

and then to $$r_2x+c=r_1y. \tag{3}$$

Now if $x=m$, $y=q_3$ be solution of (3), then

$$x=m$$
$$y=mq_2+q_3$$

is a solution of (2), and

$$\left.\begin{array}{l} y=mq_2+q_3 \\ x=(mq_2+q_3)q_1+m \end{array}\right\} \tag{4}$$

is a solution of (1).[1]

---

1. For, if $x=m$, $y=q_3$ be a solution of (3), then

$$r_2m+c=r_1q_3,$$

or $(a-r_1q_2)m+c=r_1q_3$.

$\therefore$ $c=r_1(mq_2+q_3)-am$.

Application of this value of c reduces (2) to

$$ax+r_1(mq_2+q_3)-am=r_1y,$$

or $a(x-m)=r_1[y-(mq_2+q_3)]$,

of which a solution is evidently

$$x=m$$
$$y=mq_2+q_3.$$

Application of the same value of c reduces (1) to

$$ax+r_1(mq_2+q_3)-am=by,$$

or $ax+(b-aq_1)(mq_2+q_3)-am=by$,

or $a[x-\{(mq_2+q_3)q_1+m\}]=b[y-(mq_2+q_3)]$,

of which a solution evidently is

$$y=mq_2+q_3$$
$$x=(mq_2+q_3)q_1+m.$$

One can easily see that m is the so called 'optional number' (*mati*), q_3 the quotient obtained on dividing mr_2+c by r_1, and that the solution (4) is the same as obtained by reducing the chain

$$\begin{matrix} q_1 \\ q_2 \\ m \\ q_3 \end{matrix}$$

the successive steps of reduction of the chain being

$$\begin{matrix} q_1 & q_1 & (mq_2+q_3)q_1+m \\ q_2 & mq_2+q_3 & mq_2+q_3 \\ m & m & \\ q_3 & & \end{matrix}$$

Method 2. Proceeding with the equation

$$ax+c=by \quad (1)$$

and mutually dividing a and b ($b>a$) up to the second quotient as before, we get

$$\begin{array}{l} a)\ b\ (q_1 \\ \overline{\quad r_1)\ a\ (q_2} \\ \qquad \overline{r_2} \end{array}$$

Since from the mutual division,

$$b=aq_1+r_1$$

the equation (1) becomes

$$ax+c=(aq_1+r_1)y,$$

or $x=q_1y+x_1$, where $ax_1=r_1y-c$

$$\therefore \quad ax_1+c=r_1y \quad (2)$$

where $\quad x=q_1y+x_1. \quad (3)$

And since from the mutual division

$$a=r_1q_2+r_2$$

the equation (2) becomes

$$(r_1q_2+r_2)x_1+c=r_1y,$$

or $q_2x_1+y_1=y$ where $r_1y_1=r_2x_1+c$.

$$\therefore \quad r_2x_1+c=r_1y_1 \qquad (4,$$

where $y=q_2x_1+y_1$. (5)

Now if $x_1=m$ (*mati*), $y=q_3$ be a trial solution of (4), then

$$\left.\begin{array}{lll} & (5) \text{ gives} & y=q_1m+q_3 \\ \text{and} & (3) \text{ gives} & x=q_1(q_2m+q_3)+m. \end{array}\right\} \qquad (6)$$

Hence a solution of equation (1) is given by (6).

Note. When the mutual division is carried to more than two quotients, the proof is similar. See Datta and Singh, *History of Hindu Mathematics*, Part II, pp. 95 ff.

CHAPTER III

KĀLAKRIYĀ

OR

THE RECKONING OF TIME

[The aim of this section is to teach theoretical astronomy as far as the determination of true positions of the planets is concerned.]

TIME DIVISIONS AND CIRCULAR DIVISIONS

वर्षं [1]द्वादश मासास्त्रिंशद्दिवसो[2] भवेत् स मासस्तु ।
षष्टिर्नाड्यो दिवसः षष्टिश्च[3] विनाडिका नाडी ॥ १ ॥

गुर्वक्षराणि षष्टिर्विनाडिकार्क्षी, षडेव वा प्राणाः ।
एवं कालविभागः, क्षेत्रविभागस्तथा भगणात् ॥ २ ॥

1. **A year consists of 12 months. A month consists of 30 days. A day consists of 60 *nāḍīs*. A *nāḍī* consists of 60 *vināḍikās* (or *vināḍīs*).**

2. **A sidereal *vināḍikā* is equal to (the time taken by a man in normal condition in pronouncing) 60 long syllables (with moderate flow of voice) *or* (in taking) 6 respirations (*prāṇas*).**

   This is the division of time. The division of a circle (*lit.* the ecliptic) proceeds in a similar manner from the revolution.[4]

---

1. So. द्वादशमासः
2. Bh. दिवसा .
3. Sū. Pa. षष्टिस्तु
4. See *supra*, i. 6, p. 13, above.

These definitions may be stated in tabular form as follows :

Table 11. The Time divisions

1 year	=12 months
1 month	=30 days
1 day	=60 *nāḍīs* (or *nāḍikās*)
1 *nāḍī* (*nāḍikā*)	= 60 *vināḍikās*
1 sidereal *vināḍikā*	=60 long syallables *or*
	= 6 respirations (*prāṇas*)

Table 12. The Circular divisions

1 revolution	= 12 signs
1 sign	=30 degrees
1 degree	=60 minutes (*kalās*)
1 minute	=60 seconds (*vikalās*)
1 second	=60 thirds (*tatparās*).

"The term *kṣetra* means *bhagola* ('Sphere of the asterisms')", writes Bhāskara I. More accurately, it means 'the circle of the asterisms' or the 'ecliptic'.

The term *kṣetra* is also used in the sense of 'a sign of the zodiac.[1] But in the present context it means a circle.

CONJUNCTIONS OF TWO PLANETS IN A *YUGA*

भगणा द्वयोर्द्वयोर्ये विशेषशेषा युगे द्वियोगास्ते ।

3. (a-b) The difference between the revolution-numbers of any two planets is the number of conjunctions of those planets in a *yuga*.

VYATIPĀTAS IN A *YUGA*

रविशशिनक्षत्रगणाः सम्मिश्राश्च व्यतीपाताः ॥ ३ ॥

3. (c-d) The (combined) revolutions of the Sun and the Moon added to themselves is the number of *Vyatīpātas* (in a *yuga*).

The phenomenon called *vyatīpāta* is of two types : (1) *Lāṭa-vyatīpāta*, and (2) *Vaidhṛta-vyatīpāta*. The former occurs when the

1. *Cf.* राशि-क्षेत्र-गृह-र्क्ष-भानि भवनं चैकार्थसंप्रत्ययाः । (*Jyotiścandrārka* by Rudradeva Sharma, N. K. Press, Lucknow, i. 165, gloss).

sum of the (tropical) longitudes of the Sun and the Moon amounts to 180 degrees and the latter when that sum amounts to 360 degrees.[1] Thus, in one combined revolution of the Sun and the Moon there occur two *vyatīpātas*.

The conception of the phenomenon of *vyatīpāta* is very old. It occurs in the *Vedāṅga-Jyautiṣa*[2] (*c.* 1400 B.C.) which states the number of *vyatīpātas* in the *yuga* of 5 years. It also occurs in the Jaina astronomical work *Jyotiṣkaraṇḍa*[3] (514 A.D.) where the rule for finding the number of *vyatīpātas* in a *yuga* of five years is formulated.

ANOMALISTIC AND SYNODIC REVOLUTIONS

स्वोच्चभगणाः स्वभगणैर्विशेषिताः[4] स्वोच्चनीचपरिवर्ताः ।

4 (a-b) The difference between the revolution-numbers of a planet and its *ucca* gives the revolutions of the planet's epicycle (in a *yuga*).

What is meant is that the difference between the revolution-numbers of a planet and its *mandocca* (apogee) gives the anomalistic revolutions of that planet; and that the difference between the revolution-numbers of a planet and its *śīghrocca* gives the synodic revolutions of that planet.

The number of the anomalistic revolutions of the Moon in a *yuga*, according to Āryabhaṭa is :

=Revolution-number of the Moon *minus* Revolution-number of the Moon's apogee
=5,77,53,336—4,88,219
=5,72,65,117.

The period of one anomalistic revolution of the Moon is, likewise, equal to 1,57,79,17,500/5,72,65,117, *i e.*, 27·55459 days, or $27^d\ 13^h\ 18^m\ 36^s.6$

---

1. See *Khaṇḍa-khādyaka*, i. 25.

2. vs. 19. The date of the *Vedāṅga-Jyautiṣa* may be derived from the position of the summer solstice (*viz.* the first point of the *nakṣatra* Dhaniṣṭhā) mentioned in that work.

3. *Gathās* 291-93. For details, see *Vedāṅga-jyautiṣa*, edited with English translation and Sanskrit commentary by R. Shamasastry, Mysore, 1936, vs. 19, notes and Sanskrit commentary.

4. G. Su. वियोजिताः

approx. According to modern astronomers, it is equal to $27^d\ 13^h\ 18^m\ 33^s.1$.

The following table gives the synodic revolutions (in a *yuga*) and the synodic periods (*niraṁśa-kāla*) of the Moon and the planets according to Āryabhaṭa I, Ptolemy and the modern astronomers.

Table 13. Synodic revolutions and synodic periods

Planet	Synodic revolutions according to Aryabhaṭa I	Synodic period in days		
		Āryabhaṭa I	Ptolemy	Modern
Moon	53433336	29·53058	29·53059	29·53059
Mars	2023176	779·92125	779·9428	779·936
Śīghrocca of Mercury	13617020	115·8783	115·8786	115·877
Jupiter	3955776	398·8895	398·8864	398·884
Śīghrocca of Venus	2702388	583·8975	584·0000	583·921
Saturn	4173436	378·0859	378·0930	378·092

JOVIAN YEARS IN A *YUGA*

गुरुभगणा राशिगुणास्त्वाश्वयुजाद्या[1] गुरोरब्दाः ॥ ४ ॥

4. (c-d) The revolution-number of Jupiter multiplied by 12 gives the the number of Jovian years beginning with *Aśvayuk* (in a *yuga*).

A Jovian year is the time taken by Jupiter in passing through one sign of the zodiac. The following table gives the names by which the Jovian years are called when Jupiter passes through the various signs.

Table 14. Names of the Jovian years

Sign	Jovian year	Sign	Jovian year
1 Aries	Aśvayuk	7 Libra	Caitra
2 Taurus	Kārtika	8 Scorpio	Vaiśākha
3 Gemini	Mārgaśīrṣa	9 Sagittarius	Jyeṣṭha
4 Cancer	Pauṣa	10 Caricorn	Āṣāḍha
5 Leo	Māgha	11 Aquarius	Śrāvaṇa
6 Virgo	Phālguna	12 Pisces	Bhādrapada

1. B. गुणास्ते च युगाद्याः; Bh. राशिगुणा अश्वयुजाद्या ।

The Jovian years were named after the asterisms in which Jupiter rises heliacally in the various signs. The following table gives, according to Varāhamihira (*d.* A.D. 587),[1] the asterisms in which Jupiter is normally seen to rise heliacally in the various Jovian years.

Table 15. Asterisms in which Jupiter rises in the various Jovian years

Jovian year	Asterisms in which Jupiter rises
Aśvayuk	Revatī, Aśvinī, Bharaṇī
Kārtika	Kṛttikā, Rohiṇī
Mārgaśīrṣa	Mṛgaśirā, Ārdrā
Pauṣa	Punarvasu, Puṣya
Māgha	Āśleṣā, Maghā
Phālguna	Pūrvā Phālgunī, Uttarā Phālgunī, Hasta
Caitra	Citrā, Svātī
Vaiśākha	Viśākhā, Anurādhā
Jyeṣṭha	Jyeṣṭhā, Mūla
Āṣāḍha	Pūrvāṣāḍhā, Uttarāṣāḍhā
Śrāvaṇa	Śravaṇa, Dhaniṣṭhā
Bhādrapada	Śatabhiṣak, Pūrva Bhādrapadā, Uttarā Bhādrapadā

The above twelve-year cycle of Jupiter is taken to start at the beginning of the current *yuga* with *Aśvayuk*, because in the beginning of the *yuga* Jupiter rose heliacally in the asterism *Aśvinī*.[2]

There is another cycle of Jupiter which consists of five 12-year cycles or 60 Jovian years. The sixty years of this cycle bear the following names : [3]

1. *Cf. Bṛhat-saṁhitā* of Varāhamihira, edited with Bhaṭṭotpala's commentary by S. Dvivedī, Banaras (1895), viii. 2.

2. यस्माद्युगादावश्विन्याममरपतिगुरुरुदयशिखरिशिखरमधिरूढस्तस्मादश्वयुजाद्याः गुरोस्समाः ।

3. See *Jyotiścandrārka* by Rudradeva Sharma, N. K. Press, Lucknow, pp. 27-28 ; *Bṛhajjyotiḥsāra* compiled and translated into

1	Vijaya	31	Rudhirodgārī
2	Jaya	32	Raktākṣa
3	Manmatha	33	Krodhana
4	Durmukha	34	Kṣaya
5	Hemalamba	35	Prabhava
6	Vilamba	36	Vibhava
7	Vikārī	37	Śukla
8	Śarvarī	38	Pramoda
9	Plava	39	Prajāpati
10	Śubhakṛt	40	Aṅgirā
11	Śobhana	41	Śrīmukha
12	Krodhī	42	Bhāva
13	Viśvāvasu	43	Yuvā
14	Parābhava	44	Dhātā
15	Plavaṅga	45	Īśvara
16	Kīlaka	46	Bahudhānya
17	Saumya	47	Pramāthī
18	Sādhāraṇa	48	Vikrama
19	Virodhakṛt	49	Vṛṣa
20	Paridhāvī	50	Citrabhānu
21	Pramādī	51	Subhānu
22	Ānanda	52	Tāraṇa
23	Rākṣasa	53	Pārthiva
24	Nala or Anala	54	Vyaya
25	Piṅgala	55	Sarvajit
26	Kālayukta	56	Sarvadhārī
27	Siddhārtha	57	Virodhī
28	Raudra	58	Vikṛta
29	Durmati	59	Khara
30	Dundubhī	60	Nandana

Hindi by Surya Narayana Siddhanti, T. K. Press, Lucknow, 1972, pp. 2-3.

This cycle took a new round at the beginning of Kaliyuga. Likewise, the current Kaliyuga started with Vijaya, the first year of this cycle.

SOLAR YEARS, AND LUNAR, CIVIL AND SIDEREAL DAYS

रविभगणा रव्यब्दाः, रविशशियोगा भवन्ति शशिमासाः ।
रविभूयोगा दिवसाः, [1]क्वावर्ताश्चापि नाक्षत्राः ॥ ५ ॥

5. **The revolutions of the Sun are solar years. The conjunctions of the Sun and the Moon are lunar months. The conjunctions of the Sun and Earth are (civil) days. The rotations of the Earth are sidereal days.**

Thus we have :

Solar years in a *yuga* =43,20,000
Lunar months in a *yuga*=5,34,33,336
Civil days in a *yuga* =1,57,79,17,500
Sidereal days in a *yuga* =1,58,22,37,500

The commentators have adopted the reading *"bhāvartāścāpi nākṣatrāḥ"* in place of *"kvāvartāścāpi nākṣatrāḥ."* Bhāskara I (A.D. 629) and Raghunātha-rāja (A.D. 1597) have, however, mentioned the latter as an alternative reading. The latter, evidently, is the correct reading as it agrees with Āryabhaṭa's theory of the Earth's rotation. The word *yoga* applied to the Sun and the Earth, as Clark notes, clearly indicates that Āryabhaṭa I believed in the rotation of the Earth. Also see *infra*, ch. iv, vs. 48.

INTERCALARY MONTHS AND OMITTED LUNAR DAYS

अधिमासका युगे ते रविमासेभ्योऽधिकास्तु ये चान्द्राः ।
शशिदिवसा विज्ञेया भूदिवसोनास्तिथिप्रलयाः ॥ ६ ॥

6. **The lunar months (in a *yuga*) which are in excess of the solar months (in a *yuga*) are (known as) the intercalary months in a *yuga* ; and the lunar days (in a *yuga*) diminished by the civil days (in a *yuga*) are known as the omitted lunar days (in a *yuga*).**

---

1. Bh. (quotes), Ra. (quotes) क्वावर्ताश्चापि; others भावर्ताश्चापि

Thus, according to Āryabhaṭa I,

Intercalary months in *yuga*=5,34,33,336—5,18,40,000=15,93,336

Omitted lunar days in a *yuga*=1 60,30,C0,080—1,57,79,17,500
=2,50,82,580

DAYS OF MEN, MANES GODS AND OF BRAHMĀ

रविवर्षं मानुष्यं, तदपि त्रिंशद्गुणं भवति पित्र्यम् ।
पित्र्यं द्वादशगुणितं दिव्यं वर्षं विनिर्दिष्टम्[1] ॥ ७ ॥
दिव्यं वर्षसहस्रं [2]ग्रहसामान्यं युगं[3] द्विषट्कगुणम् ।
अष्टोत्तरं सहस्रं ब्राह्मो दिवसो ग्रहयुगानाम् ॥ ८ ॥

7. **A solar year is a year of men. Thirty times a year of men is a year of the Manes. Twelve times a year of the Manes is called a divine year (or a year of the gods).**

8. **12000 divine years make a general planetary *yuga*. 1008 (general) planetary *yugas* make a day of Brahmā.**

The statement that "thirty times a year of men is a year of the Manes" is inaccurate. The Manes are supposed to reside on the opposite side of the Moon. Since the length of a day on the Moon is equal to one lunar month or 30 lunar days of men, a year of the Manes is equal to 30 times a lunar year of men, not 30 times a solar year. The *Sūrya-siddhānta* (xiv 14) makes the statement correctly : "Of thirty lunar days is composed a lunar month, which is declared to be a day and night of the Manes."

What Āryabhaṭa means to say in the above stanza is that

a *yuga*=43,20,000 years

a day of Brahmā (or *Kalpa*)=1,008 *yugas* or 4,35,45,60,000 years.

This is in agreement with what he said in *Ā*, i. 5. (See p. 9, above)

---

1. B-E. Gh. Nī. Pa. Sū. समुद्दिष्टम्
2. F. युग for ग्रह
3. C. गुणं and F. गुण for युगं

UTSARPIṆĪ, APASARPIṆĪ, SUṢAMĀ AND DUṢṢAMĀ

उत्सर्पिणी युगार्धं पश्चादपसर्पिणी युगार्धं च ।
मध्ये युगस्य सुषमाऽऽदावन्ते दुष्षमेन्दूच्चात् ॥ ९ ॥

9. The (first) half of a *yuga* is *Utsarpiṇī* and the second half *Apasarpiṇī*. *Suṣamā* occurs in the middle and *Duṣṣamā* in the beginning and end. (The time elapsed or to elapse is to be reckoned) from the position of the Moon's apogee.[1]

This terminology is in conformity with the teachings of the Jaina canons. The time-cycle is divided there into two halves : (1) the auspicious half called *Utsarpiṇī* and (2) the inauspicious half called *Apasarpiṇī* (or *Avasarpiṇī*). The *Utsarpiṇī* is subdivided into six divisions which occur in the following sequence :

Subdivision	Division	
(1) *Duṣṣamā-duṣṣamā*		
(2) *Duṣṣamā*	*DUṢṢAMĀ*	(1)
(3) *Duṣṣamā-suṣamā*		
(4) *Suṣamā-duṣṣamā*		
(5) *Suṣamā*	*SUṢAMĀ*	(2)
(6) *Suṣamā-suṣamā*		

The *Apasarpiṇī* is also similarly subdivided into six divisions which occur in the following succession :

Subdivision	Division	
(7) *Suṣamā-suṣamā*		
(8) *Suṣamā*	*SUṢAMĀ*	(3)
(9) *Suṣamā-Duṣṣamā*		
(10) *Duṣṣamā-suṣamā*		
(11) *Duṣṣamā*	*DUṢṢAMĀ*	(4)
(12) *Duṣṣamā-duṣṣamā*		

Instead of dividing the *Utsarpiṇī* and the *Apasarpiṇī* into six divisions each, Āryabhaṭa has divided each of them into two gross divisions, *Utsarpiṇi* into *Duṣṣamā* and *Suṣamā*, and *Apasarpiṇī* into *Suṣamā* and *Duṣṣamā*. The two *Suṣamā* divisions thus fall in the middle of a *yuga* and the two *Duṣṣamā* divisions in the beginning and end of a *yuga*.

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1. Same is stated in *Vaṭeśvara-siddhānta*, *Grahagaṇita*, ch. 1, sec. 2, vs. 6.

The time elapsed is reckoned from the position of the Moon's apogee, because in the case of the Moon's apogee there is no abraded *yuga*. The *yuga* of the Moon's apogee is the same as the general planetary *yuga* of 43,20,000 years (or 1,57,79,17,500 days). In the case of the other planets, there are abraded *yugas* of smaller durations which are not to be used. The idea is that in the determination of the elapsed or unelapsed portion of a *yuga* the general planetary *yuga* has to be taken into account and not the abraded *yugas* of the planets.

Table 16. Abraded yugas and corresponding revolutions

Planet	*Yuga* (in days)	Revolutions
Sun	2,10,389	576
Moon	21,55,625	78,898
Moon's apogee	1,57,79,17,500	4,88,219
Moon's ascending node	78,89,58,750	1,16,113
Mars	13,14,93,125	1,91,402
Śīghrocca of Mercury	7,88,95,875	8,96,851
Jupiter	13,14,93,125	30,352
Śīghrocca of Venus	13,14,93,125	5,85,199
Saturn	39,44,79,375	36,641

In the *Kāśyapa-saṁhita*,[1] a Hindu work on pediatrics, too, time is classified into two categories, auspicious time (*śubhakāla*) and inauspicious time (*aśubha-kāla*). The auspicious time is called *Utsarpiṇī* and the inauspicious time *Apasarpiṇī*. But, there, the *Utsarpiṇī* is subdivided into three parts, *viz.* :

(1) *Ādi-yuga*

(2) *Deva-yuga*

(3) *Kṛta-yuga*

The *Apasarpiṇī* is also subdivided into three parts, *viz.* :

(4) *Tretā-yuga*

(5) *Dvāpara-yuga*

(6) *Kali-yuga*

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1. See *Kāśyapa-saṁhitā*, edited by Hemarāja Śarmā, Nepal Sanskrit Series, No. 1, Bombay (1938), p. 44.

Kāśyapa has evidently tried to establish an excellent compromise between the Jaina and the Hindu conceptions.

According to the orthodox Hindu conception, a general planetary *yuga* is divided into 4 smaller *yugas* called *Kṛta*, *Tretā*, *Dvāpara* and *Kali*. The lengths of these *yugas* and the measures of righteousness in them are supposed to be in the ratio of 4 : 3 : 2 : 1. The defect in the Hindu conception is that Kaliyuga, which is marked by one-quarter of righteousness, is abruptly followed by Kṛtayuga, which is marked by four quarters of righteousness.

The commentators of the *Āryabhaṭīya* have taken full liberty in interpreting the text. The interpretations given by Bhāskara I (A.D. 629), Sūryadeva (*b.* A.D. 1191) and Nīlakaṇṭha (A.D. 1500) are quite arbitrary and different from one another.

Two similar terms *Avarohiṇī* and *Ārohiṇī* are found to occur in Hindu works on horoscopy. According to Gārgī, so long as a planet moves from its apogee to perigee the *Daśā* or *Antardaśa* of that planet is called *Avarohiṇī* and so long as a planet moves from its perigee to its apogee, the *Daśā* or *Antardaśa* of that planet is called *Ārohiṇī*. Varāhamihira, too, says the same thing. (See *Bṛhajjātaka*, viii. 6, and Bhaṭṭotpala's commentary thereon.)

DATE OF ĀRYABHAṬA I

षष्ट्यब्दानां षष्टिर्यदा व्यतीतास्त्रयश्च युगपादाः ।
त्र्यधिका विंशतिरब्दास्तदेह मम जन्मनोऽतीताः ॥ १० ॥

10. When sixty times sixty years and three quarter *yugas* (of the current *yuga*) had elapsed, twentythree years had then passed since my birth.

This stanza mentions the epoch when 3600 years had elapsed since the beginning of the current Kaliyuga. Since

$$3600 \text{ years} = \frac{1577917500}{1200} \text{ or } 1314931{\cdot}25 \text{ days},$$

this epoch corresponds to mean noon at Ujjayinī, Sunday, March 21, 499 A.D. At this time Āryabhaṭa I was exactly 23 years of age. **Āryabhaṭa I was therefore born on March 21, 476 A.D.**

The object of specifying the year 3600 of the Kali era, according to the commentators of the *Āryabhaṭīya,* was to show that at that time the mean positions of the planets computed from the parameters given in the *Daśagītikā-sūtra* did not require any correction, and that to the mean longitudes computed for a subsequent date a *bīja* correction was necessary. Such a correction is given by Lalla, who belongs to the school of Āryabhaṭa I.

Table 17. Bīja corrections according to Lalla

(Epoch of zero correction, though really Śaka 421, is taken as Śaka 420 for facility of computation)

Planet	*Bīja* correcton to the mean longitudes of the planets per annum in terms of minutes of arc
Moon	— 25/250
Moon's apogee	— 114/250
Moon's asc. node	— 96/250
Mars	+ 48/250
Mercury	+420/250 or +430/250
Jupiter	— 47/250
Venus	—153/250
Saturn	+ 20/250

The Kali year 3600 (or Śaka year 421) was, according to Sūryadeva (*b.* A.D. 1191), Raghunātha-rāja (A.D. 1597), Viśvanātha[1] (A.D. 1629), and also according to the author of the *Vākya-karaṇa* (*c.* A.D. 1300), the time when the precession of the equinoxes was also zero.

According to another view, A.D. 522 (corresponding to the Śaka year 444) was the epoch of zero correction to the mean longitudes of the planets calculated from the parameters stated in the *Daśagītikā-sūtra*. Astronomers holding this view have prescribed the following *bīja* corrections.

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1. See his comm. on *Makaranda-sāraṇī*, Bombay (1935), p. 84.

Table 18. Bīja correction according to Haridatta[1] and Deva[2] (689 A.D.)
(Epoch of zero correction being Śaka 444 or A.D. 522)

Planet	*Bīja* correction to the mean longitudes of the planets per annum in terms of minutes of arc
Moon	— 25/235
Moon's apogee	—114/235
Moon's asc. node	— 96/235
Mars	+ 45/235
Mercury	+420/235
Jupiter	— 47/235
Venus	—153/235
Saturn	+ 20/235

Table 19. Another bīja[3] correction with epoch at Śaka 444 (A.D. 522)

Planet	*Bīja* correction per annum in terms of minutes of arc
Moon	— 25/235
Moon's apogee	—114/235
Moon's asc. node	— 96/235
Mars	+ 50/235
Mercury	+430/235
Jupiter	— 50/235
Venus	—160/235
Saturn	+ 21/235

1. See *Grahacāra-nibandhana* of Haridatta, ed. K.V. Sarma, p. 25, vss. 17-18.

2. *KR*, i. 16-18.

3. See *Grahacāra-nibandhana*, pp. 25-26, vss. 19-22. Sūryadeva, in his comm. on *LMā*, i. 1-2, calls this correction 'traditional' (*sampradāya-siddha*); but for Venus he gives —180/235 mins. in place of —160/235 mins.

Brahmadeva (A.D. 1092), who wrote his calendrical work *Karaṇa-prakāśa* on the basis of Lalla's *Śiṣya-dhī-vṛddhida*, Bhoja (A.D. 1042), who wrote the calendrical work *Rāja-mṛgāṅka*, Gaṇeśa Daivajña (A.D. 1520), the author of the calendrical work *Grahalāghava*, Mañjula (A.D. 932), the author of the *Laghu-mānasa*, and some followers of the *Khaṇḍa-khādyaka* have regarded Śaka 444 (A.D. 522) as the epoch when the precession of the equinoxes was zero.

Some astronomers of Kerala have associated both Śaka 421 and Śaka 444 with the life of Āryabhaṭa and have called them *Bhaṭābda* ('the years associated with Āryabhaṭa'). The Kerala astronomer Haridatta (*c*. A.D. 683), the alleged author of the *Śakābda* correction, has, as remarked by Nīlakaṇṭha in his commentary on the *Āryabhaṭīya* (rather in surprise), interpreted the above stanza in a different way, *viz.* :

> "When sixty times sixty years and three quarter *yugas* had elapsed, twentythree years of my age have passed since then."

This means that Āryabhaṭa was born in Śaka 421 and wrote the *Āryabhaṭīya* in Śaka 444. But no commentator of the *Āryabhaṭīya* has interpreted the above stanza in this way, and T.S. Kuppanna Śāstrī has rightly called it a "wrong interpretation". Another Kerala astronomer (probably Jyeṣṭhadeva), the author of the *Dṛkkaraṇa* (A.D. 1603), an astronomical manual in Malayalam, has actually stated that Āryabhaṭa was born in Śaka 421 and that he wrote the *Āryabhaṭīya* in Śaka 444. This is, according to T.S. Kuppanna Śāstrī, a "mistaken impression".

According to the commentators Sūryadeva (*b.* A.D. 1191) Parameśvara (A. D. 1431) and Nīlakaṇṭha (A. D. 1500), the Kali year 3600 (corresponding to Śaka 421), besides being the epoch of zero correction, indicates the time of composition of the *Āryabhaṭīya*. K. Sāmbaśiva Śāstrī, W.E. Clark, and Baladeva Miśra, too, hold the same opinion.

BEGINNING OF THE YUGA, YEAR, MONTH, AND DAY

युगवर्षमासदिवसाः समं प्रवृत्तास्तु[1] चैत्रशुक्लादेः ।
कालोऽयमनाद्यन्तो ग्रहभैरनुमीयते क्षेत्रे ॥ ११ ॥

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1. D. प्रवृत्ताश्च

11. The *yuga*, the year, the month, and the day commenced simultaneously at the beginning of the light half of Caitra.[1] This time, which is without beginning and end, is measured with the help of the planets and the asterisms on the Celestial Sphere.

What is meant is that time is endless and has no beginning or end, but for practical purposes it is measured by means of the *yuga*, the year, the month, and the day etc., which are defined on the basis of the positions of the planets and the asterisms in the sky, in the same way as length is measured by means of the units of length.

The commentator Bhāskara I provides an alternative interpretation for *grahabhair anumīyate*. "Others", writes he, "interpret *grahabhair anumīyate* in a different way, *viz.*, the beginning or end of time is defined with the help of the planets and the asterisms." The commentator Someśvara, too, interprets the above stanza in the same way. Writes he : "The *yuga*, the year, the month, and the day started simultaneously at the beginning of Caitra, *i.e.*, at the beginning of the light fortnight of Caitra when half the Sun had risen (above the horizon at Laṅkā). Then it would mean that time has a beginning ; to contradict this, Āryabhaṭa says : 'This time is without beginning or end', *i.e.*, the time which we have referred to as *yuga* etc. and which started at the beginning of the light half of Caitra, has neither a beginning nor an end. It is only for the use of the people that its beginning and end are defined. How are its beginning and end defined for use ? Āryabhaṭa says : 'These are defined by (the positions of) the planets and the asterisms'. As for example, the beginning of the *yuga* is defined as the time when all the planets are simultaneously on the horizon (at Laṅkā) at the first point of Aries."

Since the calculation of the positions of the planets is ultimately aimed at the determination of time, the nomenclature 'the reckoning of time' given to the present chapter is highly significant.

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1. *Cf. BrSpSi*, i. 4 ; *MSi*, i. 5 ; *SiŚe*, i. 10 ; *SiŚi*, I. i. 15.

EQUALITY OF THE LINEAR MOTION OF THE PLANETS

षष्ट्या सूर्याब्दानां प्रपूरयन्ति ग्रहा भपरिणाहम्[1] ।
दिव्येन नभःपरिधिं समं भ्रमन्तः स्वकक्ष्यासु ॥ १२ ॥

12. The planets moving with equal linear velocity in their own orbits[2] complete (a distance equal to) the circumference of the sphere of the asterisms in a period of 60 solar years, and (a distance equal to) the circumference of the sphere of the sky in a *yuga*.[3]

That is, a planet moves through a distance of 17,32,60,008 *yojanas* in 60 solar years and a distance of 1,24,74,72,05,76,000 *yojanas* in 43,20,000 solar years. Since there are 1,57,79,17,500 days in 43,20,000 solar years, it follows that the mean daily motion of a planet, according to Āryabhaṭa, is

$$\frac{12474720576000}{1577917500} \textit{ yojanas}$$

or 7905·8 *yojanas* approx.

For further details, the reader is referred to our notes on *Ā*. i.6 (pp. 13-15, above).

CONSEQUENCE OF EQUAL LINEAR MOTION OF THE PLANETS

मण्डलमल्पमधस्तात् कालेनाल्पेन पूरयति चन्द्रः ।
उपरिष्टात् सर्वेषां महच्च महता शनैश्चारी ॥ १३ ॥

13. The Moon completes its lowest and smallest orbit in the shortest time; Saturn completes its highest and largest orbit in the longest time.[4]

For the lengths of the orbits of the planets, see *supra*, i. 6 (p. 13, above).

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1. C. परिमाणम्
2. *Cf. PSi*, xiii. 39 (c-d).
3. *Cf. BrSpSi*, xxi. 12.
4. *Cf. PSi*, xiii. 41 ; *SūSi*, xii. 76-77 ; *SiŚi*, I, i. 5. 27 (c-d).

NON-EQUALITY OF THE LINEAR MEASURES OF THE CIRCULAR DIVISIONS

अल्पे हि मण्डलेऽल्पा महति महान्तश्च राशयो ज्ञेयाः ।
अंशाः कलास्तथैवं[1] विभागतुल्याः स्वकक्ष्यासु ॥ १४ ॥

14. (The linear measures of) the signs are to be known to be small in small orbits and large in large orbits ;[2] so also are (the linear measures of) the degrees, minutes, etc. The circular division is however, the same in the orbits of the various planets.

The first part of the above statement would be self-evident from the following table which gives the lengths in *yojanas* of one sign, one degree and one minute in the orbits of the various planets. The planets have been arranged, for the sake of convenience, in the order of increasing orbits.

Table 20. Lengths in yojanas of the circular divisions of the orbits

Orbit of	Lengths in *yojanas* of		
	1 sign	1 degree	1 minute
Moon	18000	600	10
Śīghrocca of Mercury	57956	1932	32
Śīghrocca of Venus	148035	4935	82
Sun	240639	8021	134
Mars	452608	15087	251
Jupiter	2854178	95139	1586
Saturn	7092874	236429	3940

1. C. E. Sū. तथैव ; C. Sū. add च here.

2. *Cf. PSi*, xiii. 40 ; *SuSi*, xii. 75 (c-d) ; *SiŚi*, I, i. 5. 27 (b).

The second part of the statement means that in the orbits of all the planets one sign is equal to 1/12 of the orbit, one degree is equal to 1/30 of a sign, one minute is equal to 1/60 of a degree, and so on.

The non-equality of the linear measures of the circular divisions in the orbits of the various planets implies that although the planets have equal linear velocity, their angular velocities are different. The following table gives the mean angular velocities of the planets according to Āryabhaṭa I.[1]

Table 21. Mean angular velocities of the planets

Planet	Mean angular velocity per day
Sun	59′ 8″
Moon	790′ 35″
Moon's apogee	6′ 41″
Moon's ascending node	3′ 11″
Mars	31′ 26″
Śīghrocca of Mercury	4° 5′ 32″
Jupiter	4′ 59″ or 5′ 00″ approx.
Śīghrocca of Venus	1° 36′ 8″
Saturn	2′ 0″

RELATIVE POSITIONS OF ASTERISMS AND PLANETS

भानामधः शनैश्चरसुरगुरुभौमार्कशुक्रबुधचन्द्राः ।
[2]एषामधश्च भूमिर्मेधीभूता खमध्यस्था ॥ १५ ॥

15. **(The asterisms are the outermost). Beneath the asterisms lie (the planets) Saturn, Jupiter, Mars, the Sun, Venus, Mercury,**

1. See *SiDVr*, I, i. 40-41.
2. All others than Bh., तेषां for एषां

and the Moon (one below the other) ; and beneath them all lies the Earth like the hitching peg in the midst of space.[1]

LORDS OF THE HOURS AND DAYS

सप्तैते होरेशाः शनैश्चराद्या यथाक्रमं शीघ्राः ।
शीघ्रक्रमाच्चतुर्था भवन्ति सूर्योदयाद्[2] दिनपाः ॥ १६ ॥

16. **The (above-mentioned) seven planets beginning with Saturn, which are arranged in the order of increasing velocity, are the lords of the successive hours. The planets occurring fourth in the order of increasing velocity are the lords of the successive days, which are reckoned from sunrise (at Laṅkā).**[3]

That is to say, the lords of the twenty-four hours (the hours being reckoned from sunrise at Laṅkā) are :

> Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Saturn, Jupiter, and Mars, respectively ;

and the lords of the seven days are :

> Saturn, Sun, Moon, Mars, Mercury, Jupiter, and Venus, respectively.

The lord of a day is the lord of the first hour of that day, the day being measured from sunrise at Laṅkā.

It is to be noted that the lords of the hours and the days are to be reckoned from sunrise at Laṅkā (and not from sunrise at the local place). Since Āryabhaṭa I mentions, in the above rule, sunrise without specifying that it refers to Laṅkā, Brahmagupta finds occasion to criticise him.

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1. *Cf. PSi*, xiii. 39 ; *BrSpSi*, xxi. 2 ; *SiŚe*, xv. 70 ; *SiŚi*, II. iii. 2.

2. D. सूर्योदये

3. *Cf. PSi*, xiii. 42 ; *SuSi*, xii. 78-79.

Writes he :

> "The statement of Āryabhaṭa, *viz.*, 'Reckoned from sunrise, the planets occurring fourth (in the order of increasing velocity) are the lords of the successive days' is not true, because he has himself declared sunset at Siddhapura when it is sunrise at Laṅkā."[1]

On this criticism, Brahmagupta's commentator Pṛthūdaka comments :

> "This is a phantom of a defect, for, in the ***Daśagītikā***, Āryabhaṭa has (already) said—'from sunrise ***at Laṅkā***'."

As regards the first day of the week cycle, it is perhaps implied in the above rule that it was Saturday. Vaṭeśvara (A.D. 904) is the only Hindu mathematician who supposed that the world-order commenced on Saturday. He has criticised Brahmagupta for starting the *Kalpa* on Sunday :

> "The lords of the hours, days, months and years have been stated by Brahmā to succeed one another in the order of increasing velocity beginning with Saturn and not with the Sun. Even the order of the planets are not known to him."[2]

MOTION OF THE PLANETS EXPLAINED THROUGH ECCENTRIC CIRCLES

कक्ष्याप्रतिमण्डलगा भ्रमन्ति सर्वे ग्रहाः स्वचारेण ।
मन्दोच्चादनुलोमं प्रतिलोमं चैव शीघ्रोच्चात् ॥ १७ ॥

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1. सूर्योदयाच्चतुर्था दिनपा यदुवाच तदसदार्यभटः ।
   लङ्कोदये यतोऽर्कस्यास्तमयं प्राह सिद्धपुरे ॥
   (*BrSpSi*, xi. 12)

2. शीघ्रक्रमान्निरुक्ता होरादिनमासवर्षपा धात्रा ।
   मन्दादेर्नार्कादेर्वेत्ति न वा तत्स्वरूपमपि ।।
   VaSi*, *Grahagaṇita, **ch. 1, sec. 10, vs. 9**

कक्ष्यामण्डलतुल्यं स्वं स्वं प्रतिमण्डलं भवत्येषाम् ।
प्रतिमण्डलस्य मध्यं घनभूमध्यादतिक्रान्तम् ॥ १८ ॥
प्रतिमण्डलभूविवरं व्यासार्धं स्वोच्चनीचवृत्तस्य ।

17. (The mean planets move on their orbits and the true planets on their eccentric circles). All the planets, whether moving on their orbits (*kakṣyā-maṇḍala*) or on the eccentric circles (*prati-maṇḍala*), move with their own (mean) motion, anticlockwise from their apogees and clockwise from their *śīghroccas*.

18. The eccentric circle of each of these planets is equal to its own orbit, but the centre of the eccentric circle lies at a distance from the centre of the solid Earth.

19. (a-b) The distance between the centre of the Earth and the centre of the eccentric circle is (equal to) the semi-diameter of the epicycle (of the planet).

MOTION OF PLANETS EXPLAINED THROUGH EPICYCLES

वृत्तपरिधौ ग्रहास्ते मध्यमचाराद्[1] भ्रमन्त्येव[2] ॥ १९ ॥
यः शीघ्रगतिः स्वोच्चात् प्रतिलोमगतिः स्ववृत्तकक्ष्यायाम् ।
अनुलोमगतिर्वृत्ते मन्दगतिर्यो ग्रहो भवति[3] ॥ २० ॥

19. (c-d) All the planets undoubtedly move with mean motion on the circumference of the epicycles.

20. A planet when faster than its *ucca* moves clockwise on the circumference of its epicycle and when slower than its *ucca* moves anticlockwise on its epicycle.[4]

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1. B. D. E. Bh. Gh. So. Sū. मध्यमचारात् ; others मध्यचारं
2. Bh. भ्रमन्त्येवम्
3. B-E, Gh. Nī. Pa. Sū. भ्रमति
4. *Cf. BrSpSi*, xxi. 25-26 ; *ŚiDVṛ*, II. i. 12 (a-b) ; *SiŚe*, xvi. 5 ; *SiŚi*, II, v. 30.

What is meant is that a planet moves clockwise on its *manda* epicycle and anticlockwise on its *śīghra* epicycle.

According to the commentator Bhāskara I, verse 20 relates to the determination of the true daily motion, retrograde or direct. He has interpreted this verse as follows :

> "When the *śīghragatiphala* (*śīghra*-motion-correction) is negative but numerically greater than the true-mean motion, their difference gives the retrograde motion ; and when *śīghragatiphala* (*śīghra*-motion-correction) is negative but numerically less than the true-mean motion, their difference gives the direct motion. This latter motion when less than mean motion is called slow motion (*mandagati*)."

Following Bhāskara I, the commentator Someśvara, too, interprets the verse in the same way.

Evidently, both Bhāskara I and Someśvara have misunderstood the text.

MOTION OF EPICYCLES

अनुलोमगानि मन्दात् शीघ्रात् प्रतिलोमगानि वृत्तानि ।
कक्ष्यामण्डललग्नस्ववृत्तमध्ये ग्रहो मध्यः ॥ २१ ॥

21. **The epicycles move anticlockwise from the apogees and clockwise from the *śīghroccas*. The mean planet lies at the centre of its epicycle, which is situated on the (planet's) orbit.**

What is meant is that the *manda* epicycles move anticlockwise from the apogees and the *śīghra* epicycles move clockwise from the *śīghroccas*.

ADDITION AND SUBTRACTION OF MANDAPHALA AND ŚIGHRAPHALA

क्षयधन[1]धनक्षयाः स्युर्मन्दोच्चाद् व्यत्ययेन[2] शीघ्रोच्चात् ।

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1. E. F. Ni. Pa. ऋणधन
2. मन्दोच्चादेवमेव var. recorded by Bh. (in his com. on this verse)

22. (a-b) The corrections from the apogee (for the four anomalistic quadrants) are respectively minus, plus, plus, and minus. Those from the *śīghrocca* are just the reverse.[1]

In the time of Āryabhaṭa I, the Rsines of the arcs ($\triangleright 90°$) were obtained by the application of the following formulae :

$$\text{Rsin}\ (90°+\theta)=\text{Rsin}\ 90°-\text{Rvers}\ \theta$$

$$\text{Rsin}\ (180°+\theta)=\text{Rsin}\ 90°-\text{Rvers}\ 90°-\text{Rsin}\ \theta$$

$$\text{Rsin}\ (270°+\theta)=\text{Rsin}\ 90°-\text{Rvers}\ 90°-\text{Rsin}\ 90°+\text{Rvers}\ \theta,$$

where $\theta<90°$.

Suppose that a planet lies in the fourth *manda* anomalistic quadrant and that the *manda* anomaly is $270°+\theta$. Then

$$\text{Rsin}\ (270°+\theta)=\text{Rsin}\ 90°-\text{Rvers}\ 90°-\text{Rsin}\ 90°+\text{Rvers}\ \theta,$$

so that

$$\text{Rsin}\ (90°-\theta)=-\text{Rsin}\ 90°+\text{Rvers}\ 90°+\text{Rsin}\ 90°-\text{Rvers}\ \theta,$$

or

$$\textit{Mandakendrabhujajyā}=-\text{Rsin}\ 90°+\text{Rvers}\ 90°+\text{Rsin}\ 90°-\text{Rvers}\ \theta. \quad \ldots\ldots(1)$$

Now, multiplying both sides of (1) by the planet's *manda* epicycle and dividing by 360, we get

Correction from the apogee (*mandaphala*)

= —correction for the first quadrant+correction for the second quadrant+correction for the third quadrant—correction for the fourth quadrant,

whence it is clear that the corrections for the first, second, third, and fourth quadrants are —, +, +, and —, respectively.

The same can be seen to be true when the planet is in the other anomalistic quadrants.

In the case of the *śīghraphala*, the correction for the four quadrants are of the contrary signs, because the *mandakendra* and the *śīghrakendra* are defined contrarily :

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1. *Cf. BrSpSi*, ii. 16 (a-b).

mandakendra = longitude of planet—longitude of planet's apogee.

śīghrakendra = longitude of planet's *śīghrocca*—longitude of planet.

The law of addition and subtraction of the *mandaphala* and *śīghraphala* in the four quadrants is mentioned also by Bhāskara I, (A.D. 629), Brahmagupta (A.D. 628) and Śrīpati (*c.* A.D. 1039), but it was more convenient to apply the *mandaphala* as obtained by the formula :

$$mandaphala = \frac{\text{Rsin } \theta \times manda \text{ epicycle}}{360},$$

(θ being the planet's *mandakendra* reduced to *bhuja*)

negatively or positively, according as the *mandakendra* was less than or greater than 180°, and the *śīghraphala* as obtained by the formula

$$śīghraphala = \frac{\text{Rsin } \theta \times śīghra \text{ epicycle}}{360} \times \frac{R}{H},$$

(θ being the planet's *śīghrakendra* reduced to *bhuja* and H the planet's *śīghrakarṇa*)

positively or negatively, according as the *śīghrakendra* was less than or greater than 180°. And so the Hindu astronomers have generally adopted these latter rules.

A SPECIAL PRE-CORRECTION FOR THE SUPERIOR PLANETS

शनिगुरुकुजेषु मन्दादर्धमृणं धनं भवति पूर्वं[1] ॥ २२ ॥

22. (c-d) In the case of (the superior planets) Saturn, Jupiter and Mars, first apply the *mandaphala* **negatively or positively (as the case may be).**

Here the following rule is implied :

In the case of Saturn, Jupiter and Mars, first apply half the *mandaphala* to the mean longitude of the planet negatively or positively, according as the *mandakendra* is less than or greater than 180°.

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1. मन्देऽर्धमृणं धनं भवति पूर्वम्, var. recorded by Bhāskara I in his comm.

This pre-correction is meant only for the superior planets—Mars, Jupiter and Saturn. It should not be applied to the inferior planets, Mercury and Venus. (See full rules given below).

PROCEDURE OF MANDAPHALA AND ŚIGHRAPHALA CORRECTIONS FOR SUPERIOR PLANETS

मन्दोच्चाच्छीघ्रोच्चादर्धमृणं धनं ग्रहेषु मन्देषु ।
मन्दोच्चात् स्फुटमध्याः शीघ्रोच्चाच्च स्फुटा ज्ञेयाः ॥ २३ ॥

23. Apply half the *mandaphala* and half the *śīghraphala* to the planet and to the planet's apogee negatively or positively (as the case may be). The mean planet (then) corrected for the *mandaphala* (calculated afresh from the new *mandakendra*) is called the true-mean planet and that (true-mean planet) corrected for the *śīghraphala* (calculated afresh) is known as the true planet.[1]

This rule may be stated fully as follows :

Apply half the *mandaphala* to the mean longitude of the planet negatively or positively, according as the *mandakendra* is less than or greater than 180° and to the longitude of the planet's apogee reversely. Then apply half the *śīghraphala* to the corrected longitude of the planet's apogee negatively or positively, according as the *śīghrakendra* is less than or greater than 180°.

Then calculate the *mandaphala* afresh and apply the whole of it to the (original) mean longitude of the planet negatively or positively, according as the *mandakendra* is less than or greater than 180° : this would give the true-mean longitude of the planet. Then calculate the *śīghraphala* again and apply the whole of it to the true-mean longitude of the planet positively or negatively, according as the *śīghrakendra* is less than or greater than 180° : this would give the true longitude of the planet.

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1. *Cf. MBh*, iv. 40-43 ; *LBh*, ii. 33-37 (a-b) ; *ŚiDVṛ*, I, iii. 4-7.

MANDAPHALA AND ŚĪGHRAPHALA CORRECTIONS FOR INFERIOR PLANETS

शीघ्रोच्चादर्धोनं कर्तव्यमृणं धनं स्वमन्दोच्चे ।
स्फुटमध्यौ तु भृगुबुधौ[1] सिद्धान्मन्दात् स्फुटौ भवतः ॥ २४ ॥

24. (In the case of Mercury and Venus) apply half the *śīghraphala* negatively or positively to the longitude of the planet's apogee (according as the *śīghrakendra* is less than or greater than 180°). From the corrected longitude of the planet's apogee (calculate the *mandaphala* afresh and apply it to the mean longitude of the planet ; then) are obtained the true-mean longitudes of Mercury and Venus. (The *śīghraphala*, calculated afresh, being applied to them), they become true (longitudes).[2]

The old *Sūrya-siddhānta* applied the *mandaphala* and *śīghraphala* corrections in the following order :

(a) For obtaining the true longitude of the planet's apogee :

1. Half *śīghraphala* to the longitude of the planet's apogee (reversely).
2. Half *mandaphala* to the corrected longitude of the planet's apogee (reversely).

(b) For obtaining the true longitude of the planet :

3. Entire *mandaphala* (calculated from the corrected longitude of the planet's apogee) to the mean longitude of the planet.
4. Entire *śīghraphala* to the corrected mean longitude (called true-mean longitude) of the planet.

But instead of applying a pre-correction in the case of the superior planets (as done by Āryabhaṭa), it prescribed an impirical correction (called the fifth correction) in the case of the inferior planets.[3]

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1. A. शुक्रबुधौ

2. *Cf. MBh*, iv. 44; *LBh*, ii. 37(c-d)-39; *ŚiDVr*, I, iii. 8.

3. See *PSi*, xvi. 17-22. Also see K.S. Shukla, The *Pañcasiddhāntikā* of Varāhamihira (I), *IJHS*, vol. 9, no. 1, pp. 69-71.

It seems that the procedure used by the author of the old *Sūrya-siddhānta* did not lead to accurate results and that Āryabhaṭa's method was an improvement.

DISTANCE AND VELOCITY OF A PLANET

भूताराग्रहविवरं व्यासार्धहृतः[1] स्वकर्णसंवर्गः ।
कक्ष्यायां ग्रहवेगो यो भवति स मन्दनीचोच्चे ।। २५ ।।

[इति कालक्रियापादः समाप्तः ।[2]]

25. **The product of the *mandakarṇa* and the *śīghrakarṇa* when divided by the radius gives the distance between the Earth and the planet.**[3]

The velocity of the (true) planet moving on the (*śīghra*) epicycle is the same as the velocity of the (true-mean) planet moving in its orbit (of radius equal to the *mandakarṇa*).

Āryabhaṭa and his followers take the distance between the Earth and a planet as equal to

$$\frac{mandakarṇa \times śīghrakarṇa}{R},$$

the *mandakarṇa* and the *śīghrakarṇa* being obviously the *karṇas* obtained in the last two operations.

The *Sūrya-siddhānta*[4] takes the distance between the Earth and a planet as equal to

$$\frac{mandakarṇa + śīghrakarṇa}{2}.$$

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1. D. E. F. व्यासार्धहृतं
2. A. इति कालक्रियापादः ; (E. om. इति) ; F. इति कालक्रिया समाप्तः
3. The same rule occurs in *MBh*, vi. 48 ; *LBh*, vii, 8.
4. *SūSi*, vii. 14.

Āryabhaṭa takes the orbit of the true-mean planet as equal to the *mandakarṇa*. Hence the rule in the second half of the stanza.

The commentator Sūryadeva interprets the second half of the verse as meaning :

> "The velocity of the (true) planet in the (*śīghra*) epicycle is the same as the velocity of the planet in the orbit constructed with radius equal to the distance of the planet from the Earth."

What he means to say is that the velocity of the (true) planet moving on the (*śīghra*) epicycle is equal to the true-mean velocity.

CHAPTER IV

GOLA OR THE CELESTIAL SPHERE

[In order to demonstrate the motion of the heavenly bodies, the Hindu astronomers make use of spheres constructed by means of circles made of flexible wooden sticks or bamboo strips. These are called *Gola* and correspond to the Celestial Sphere of modern astronomy. The *Gola* which is supposed to be centred at the Earth's centre is called *Bhagola* ('Sphere of the asterisms'). It is used to demonstrate the motion of the Sun, the Moon and the planets in their orbits. The principal circles of this sphere are : (1) the celestial equator, (2) the ecliptic, (3) the orbits of the Moon and the planets, and (4) the day-circles, etc. The *Gola* which is supposed to be centred at the observer is called *Khagola* ('Sphere of the sky'). It is fixed in position and is used to demonstrate the diurnal motion of the heavenly bodies ; the principal circles of this sphere are : (1) the horizon, (2) the meridian, (3) the prime vertical, and (4) the six o'clock circle, etc. In the present Section, Āryabhaṭa aims at teaching spherical astronomy. He begins by giving a brief description of the *Bhagola* and the *Khagola* and then, with their help, demonstrates the motion of the heavenly bodies.]

1. *Bhagola*

POSITION OF THE ECLIPTIC

मेषादेः कन्यान्तं सममुदगपमण्डलार्धमपयातम् ।
तौल्यादेर्मीनान्तं शेषार्धं दक्षिणेनैव ॥ १ ॥

1. **One half of the ecliptic, running from the beginning of the sign Aries to the end of the sign Virgo, lies obliquely inclined (to the equator) northwards. The remaining half (of the ecliptic) running from the beginning of the sign Libra to the end of the sign Pisces, lies (equally inclined to the equator) southwards.**[2]

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1. D. जूकादेः

2. *Cf. BrSpSi*, xxi. 52 ; *ŚiDVṛ*, II, ii. 7 ; *VSi*, *Gola*, iv. 7 ; *SiŚe*, xvi. 32 ; *SiŚi*, II, vi.12 ; *SuSi*, II, iv. 6 (a-b).

Reference to the equator without defining it shows that its position was supposed to be well known and that it was already shown on the *Bhagola.*

The word *eva,* says the commentator Bhāskara I, is superfluous and is meant to complete the *āryā* verse. In case the alternative reading *evam* is adopted, the word 'similarly' will have to be added in the beginning of the second sentence (in the translation above).

Bhāskara I thinks that the word *sama* is intended to suggest that the signs of the ecliptic are of equal measure, *i.e.*, each of 30°.

MOTION OF THE NODES, THE SUN AND THE EARTH'S SHADOW

ताराग्रहेन्दुपाता भ्रमन्त्यजस्रमपमण्डलेऽर्कश्च ।
अर्काच्च[1] मण्डलार्धे भ्रमति हि तस्मिन् क्षितिच्छाया ॥ २ ॥

2. **The nodes of the star-planets (Mars, Mercury, Jupiter, Venus and Saturn) and of the Moon incessently move on the ecliptic. So also does the Sun. From the Sun, at a distance of half a circle, moves thereon the Shadow of the Earth.[2]**

The nodes of a planet are the two points where the orbit of the planet intersects the ecliptic. The point where the planet crosses the ecliptic in its northerly course in called the 'ascending node' and the point where the planet crosses the ecliptic in its southerly course is called the 'descending node'.

MOTION OF THE MOON AND THE PLANETS

अपमण्डलस्य चन्द्रः पाताद् यात्युत्तरेण दक्षिणतः ।
कुजगुरु[3]कोणाश्चैवं शीघ्रोच्चेनापि बुधशुक्रौ ॥ ३ ॥

3. **The Moon moves to the north and to the south of the ecliptic (respectively) from its (ascending and descending) nodes. So**

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1. B. अर्कात् स (wr.)

2. *Cf. BrSpSi,* xxi. 53 ; *ŚiDVr,* II, ii. 8 ; *VSi, Gola,* iv. 8 ; *SiŚe,* xvi. 33 ; *SiŚi,* II, vi. 11.

3. B. C. F. Pa. गुरुकुज

also do the planets Mars, Jupiter and Saturn. Similar is also the motion of the *śīghroccas* of Mercury and Venus.[1]

With regard to the last statement, Pṛthūdaka (A. D. 860) says : "As much is the (celestial) latitude of Mercury or Venus at its *śīghrocca*, so much is its (celestial) latitude at the place occupied by it."[2]

This is so, writes Bhāskara II (A.D. 1150), because the revolution-number of the node (in the case of Mercury and Venus) is the sum of the revolution-numbers of the planet's node and the planet's *śīghra* anomaly (*i.e.*, *śīghrocca* minus planet.).[3]

The correct explanation, however, is that Mercury and Venus revolve round the Sun with the velocity of their *śīghroccas* and so the (celestial) latitudes of Mercury and Venus are really the latitudes of their *śīghroccas*.

The following rules are implied in the instructions of the text :

1. In the case of the Moon

$$\text{Rsin (latitude)} = \frac{\text{Rsin }(M - \Omega) \times \text{Rsin } i}{H},$$

where M and Ω are the true longitudes of the Moon and its ascending node, i the inclination of the Moon's orbit to the ecliptic, and H the Moon's true distance (called *mandakarṇa*).[4]

2. In the case of Mars, Jupiter and Saturn

$$\text{Rsin (latitude)} = \frac{\text{Rsin }(P - \Omega) \times \text{Rsin } i}{D},$$

where P and Ω are the true longitudes of the planet and its ascending node, i the inclination of the planet's orbit to the ecliptic, and D the distance of the planet from the Earth (as defined in *Kālakriyā*, 25).[5]

3. In the case of Mercury and Venus

$$\text{Rsin (latitude)} = \frac{\text{Rsin }(S - \Omega) \times \text{Rsin } i}{D},$$

where S and Ω are longitudes of the planet's *śīghrocca* and ascending

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1. *Cf. BrSpSi*, xxi. 54 ; *ŚiDVṛ*, II, ii. 9 ; *VSi*, *Gola*, iv. 9 ; *SiŚe*, xvi, 34-35 ; *SiŚi*, II, vi. 14.

2, 3. See *SiŚi*, II, *Golabandha*, 23-25 (a-b) ; and Bhāskara II's comm. on it.

4. *Cf. LBh*, iv. 8.

5. *Cf. MBh*, vi. 52-53 ; *LBh*, vii. 6-9 (a-b).

node, i the inclination of the planet's orbit to the ecliptic, and D the distance of the planet from the Earth (as defined in *Kālakriyā*, 25).[1]

These formulae are not accurate but, according to Bhāskara I, they conform to the teachings of Āryabhaṭa I.

The correct formula for the celestial latitude of a planet is :

$$\text{Rsin (latitude)} = \frac{\text{Rsin } (\text{II}-\Omega) \times \text{Rsin } i}{\text{D}}$$

where II is the heliocentric longitude of the planet.[2]

VISIBILITY OF THE PLANETS

चन्द्रोंऽशैर्द्वादशभिरविक्षिप्तोऽर्कान्तरस्थितो दृश्यः[3] ।
नवभिर्भृगुर्भृगोस्तै[4]र्द्व्यधिकैर्द्व्यधिकैर्यथाश्लक्ष्णाः ॥ ४ ॥

4. **When the Moon has no latitude it is visible when situated at a distance of 12 degrees (of time) from the Sun. Venus is visible when 9 degrees (of time) distant from the Sun. The other planets taken in the order of decreasing sizes (*viz.*, Jupiter, Mercury, Saturn, and Mars)[5] are visible when they are 9 degrees (of time) increased by two-s (*i.e.*, when they are 11, 13, 15 and 17 degrees of time) distant from the Sun.[6]**

One degree of time is equivalent to 4 minutes. Thus the Moon, when ahead of the Sun, is visible towards the west if the arc of the ecliptic joining the Sun and the Moon, takes at least 12×4 minutes in setting below the horizon ; and when behind the Sun, it is visible towards the east if the arc of the ecliptic joining the Sun and the Moon takes at least 12×4 minutes in rising above the horizon. In other words, the Moon will be visible at a place if the time-interval between sunrise and moonrise, or between sunset and moonset, amounts to 12×4 minutes or more. But this is the case when the Moon has no latitude.

"When, however, the Moon has some latitude," comments Bhāskara I, "it is visible earlier or later than when it is two *ghaṭikās*

1. *Cf. MBh*, vi. 52-53 ; *LBh*, vii. 6-9 (a-b).
2. See *BrSpSi*, ix. 9 ; *ŚiDVṛ*, I, xi. 6, 9 etc.
3. All except Bh. and So., स्थितैर्दृश्य:
4. D. भृगुश्च तैस्तै
5. See *supra*, i. 7.
6. *Cf. PSi*, xvi. 23 ; *MBh*, vi. 4 (c-d)-5 (a-b), 44-45 ; 44 ; *LBh*, vi. 5, vii. 1.

(*i.e.*, 12 degrees of time) distant from the Sun. For, when it has north latitude, the (Moon's) sphere being elevated towards the north, it is visible earlier than when it is two *ghaṭikās* distant from the Sun ; and when it has south latitude, the (Moon's) sphere being depressed towards the south, it is visible later than when it is two *ghaṭikās* distant from the Sun. That is why it is said— 'When the Moon has no latitude'. Therefore, the distance of the planet from the Sun should be taken after the visibility correction has been applied to the longitude of the planet."

The degrees of time for the heliacal visibility of the planets as given by the old *Sūrya-siddhānta*,[1] are the same as those given above. Those given by the *Vāsiṣṭha-siddhānta* summarised by Varāhamihira are : 12° for the Moon, 14° for Mars, 12° for Mercury, 15° for Jupiter, 8° for Venus and 15° for Saturn.[2]

According to the Greek astronomer Ptolemy (*c.* A. D. 100-178) the distances of the planets, when in the beginning of the sign Cancer (*i. e.*, when the equator and ecliptic are nearly parallel), from the true Sun, at which they become heliacally visible, are : for Saturn, 14°; for Jupiter 12° 45′; for Mars, 14° 30′; and for Venus and Mercury, in the west, 5° 40′ and 11° 30′, respectively. See The *Almagest*, xiii. 7.

BRIGHT AND DARK SIDES OF THE EARTH AND THE PLANETS

भूग्रहभानां गोलार्धानि स्वच्छायया विवर्णानि ।
अर्धानि यथासारं सूर्याभिमुखानि दीप्यन्ते ॥ ५ ॥

5. **Halves of the globes of the Earth, the planets and the stars are dark due to their own shadows ; the other halves facing the Sun are bright in proportion to their sizes.**[3]

The Hindu astronomers believed that the Sun was the only source of light in the universe and all other celestial bodies, which were spherical in shape, received their light from the Sun. Their conception that the stars too received light from the Sun and were half-luminous and half-dark is indeed wrong.

The next eight stanzas give a description of the Earth which occupies the centre of the *Bhagola*.

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1. See *PSi*, xvi. 23.
2. See *PSi*, xvii. 58.
3. *Cf. PSi*, xiii. 35 ; *ŚiDVṛ*, II, iii. 40 ; *SiŚe*, xviii. 14.

SITUATION OF THE EARTH, ITS CONSTITUTION AND SHAPE

वृत्तभपञ्जरमध्ये कक्ष्यापरिवेष्टितः खमध्यगतः ।
मृज्जलशिखिवायुमयो भूगोलः सर्वतो वृत्तः ॥ ६ ॥

6. **The globe of the Earth stands (supportless) in space at the centre of the circular frame of the asterisms (*i.e.*, at the centre of the *Bhagola*) surrounded by the orbits (of the planets); it is made up of water, earth, fire and air and is spherical (*lit.* circular on all sides).**[1]

The commentator Someśvara's statement that "the Earth, mother of all beings, stands 'motionless' in space" is against the teachings of Āryabhaṭa.

It is remarkable that Āryabhaṭa, unlike the other astronomers, takes the Earth as made up of four elements, *viz.*, earth, water, fire and air, only. The other astronomers take it as made up of five elements, *viz.*, earth, water, fire, air and ether.

EARTH COMPARED WITH THE *KADAMBA* FLOWER

यद्वत् कदम्बपुष्पग्रन्थिः प्रचितः समन्ततः कुसुमैः ।
तद्वद्धि सर्वसत्त्वैर्जलजैः स्थलजैश्च भूगोलः ॥ ७ ॥

7. **Just as the bulb of a *Kadamba* flower is covered all around by blossoms, just so is the globe of the Earth surrounded by all creatures, terrestrial as well as aquatic.**[2]

INCREASE AND DECREASE IN THE SIZE OF THE EARTH

ब्रह्मदिवसेन[3] भूमेरुपरिष्टाद् योजनं भवति वृद्धिः ।
दिनतुल्य[4]यैक[5]रात्र्या मृदुपचितायास्तदिह[6] हानिः ॥ ८ ॥

8. **During a day of Brahmā, the size of the Earth increases externally by one *yojana*; and during a night of Brahmā, which is as long as a day, this growth of the earth is destroyed.**[7]

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1. *Cf. PSi*, xiii. 1 ; *BrSpSi*, xxi. 2 ; *ŚiDVṛ*, II, iv. 1 ; *SiŚe*, xv. 22-23 ; *SiŚi*, II, iii. 2 (a-b) ; *Golasāra*, ii. 1.

2. *Cf. PSi*, xiii. 2 ; *ŚiDVṛ*, II, iv. 6.

3. A. E. Gh. Nī. Pa. ब्राह्मदिवसेन 4. B. D. E. तत्तुल्य

5. F. Nī. Pa. यैव 6. So. भवति for तदिह

7. The same statement occurs in *ŚiDVṛ*, II, v. 20 ; *SiŚi*, II, iii. 62.

Modern astronomers, too, believe in the growth of the Earth's size, but this growth, according to them is extremely insignificant. C. A. Young, in his *Text Book on Astronomy*, writes : "Since the earth is continually receiving meteoric matter, and sending nothing away from it, *it must be constantly growing larger:* but this growth is extremely insignificant ....It would take about 1000000000 years to accumulate a layer one inch thick over the earth's surface."

According to modern geologists, the rate of uplift of the earth varies from place to place and time to time. The minimum rate of uplift of the Himalayas is about 6 in. per century,[1] whereas the present rate of uplift of the earth in Greenland is 3 mm. per year.[2]

APPARENT MOTION OF THE STARS DUE TO THE EARTH'S ROTATION

अनुलोमगतिर्नौस्थः पश्यत्यचलं विलोमगं यद्वत् ।
अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ॥ ९ ॥

उदयास्तमयनिमित्तं नित्यं प्रवहेण वायुना क्षिप्तः ।
लङ्कासमपश्चिमगो भपञ्जरः सग्रहो भ्रमति ॥ १० ॥

9. **Just as a man in a boat moving forward sees the stationary objects (on either side of the river) as moving backward, just so are the stationary stars seen by people at Laṅkā (on the equator), as moving exactly towards the west.**

10. **(It so appears as if) the entire structure of the asterisms together with the planets were moving exactly towards the west of Laṅkā, being constantly driven by the provector wind, to cause their rising and setting.**

The theory of the Earth's rotation underlying the above passage was against the view generally held by the people and was severely criticised by Varāhamihira (*d.* A.D. 587) and Brahmagupta (628 A.D.) The followers of Āryabhaṭa I, who were unable to refute the criticism against the theory, fell in line with Varāhamihira and others of his ilk and have misinterpreted the above verses as conveying the contrary

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1. See D.N. Wadia, Geology of India, Macmillan and Company, London, 1949, p. 300 fn.

2. See Richard Foster Flint, *Glacial and Pleistocene Geology*, John Wiley and Sons, Inc., 1963, p. 256.

sense. See how the commentator Someśvara interprets the above verses :

> "Just as one seated on a boat sees the stationary objects such as trees etc. standing on the two sides of the river or sea moving in the contrary direction, in the same way those situated on the Earth rotating eastwards see the stationary stars located in the sky as moving in the opposite direction towards the west. Likewise, those living in Laṅkā see the stars as moving towards the west. Laṅkā is only a token, others also see in the same way. So, it is the Earth that moves towards the east; the stars are fixed. And that part of the circle of the asterisms which lies (at the moment) towards the east appears to rise, that which lies in the middle of the sky appears to culminate, and that which lies towards the west appears to set. Otherwise, the rising and setting of the stars is impossible." After saying all this he adds :
>
> "This is the false view. For, if the Earth had a motion, the world would have been inundated by the oceans, the tops of the trees and castles would have disappeared, having been blown away by the storm caused by the velocity of the Earth, and the birds etc. flying in the sky would never have returned to their nests. So, there exists not a single trace of the Earth's motion. Hence this stanza must be interpreted in another way (as follows) :
>
> "Just as a man seated on a boat moving forward sees the stationary objects moving in the contrary direction, in the same way the asterisms driven by the provector wind, due to their own motion, see the objects at Laṅkā as moving in the opposite direction, *i.e.*, they see the stationary Earth lying below as if it were rotating. Apparently also the asterisms rise in the east and move towards the west."

Pṛthūdaka (860 A.D.) in his commentary on the *Brāhma-sphuṭa-siddhānta*, supports Āryabhaṭa I's theory of the Earth's rotation. The followers of Āryabhaṭa I, who misinterpreted Āryabhaṭa I, were, according to him, afraid of the public opinion which was against the motion of the Earth.

It is noteworthy that the Greek astronomor Ptolemy (*c.* A. D. 100-178) holds that the Earth is stationary and does not move in any way locally.[1]

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1. See The *Almagest*, translated by R.C. Taliaferro, pp. 10-12.

DESCRIPTION OF THE MERU MOUNTAIN

मेरुर्योजनमात्रः[1] प्रभाकरो हिमवता परिक्षिप्तः ।
नन्दनवनस्य मध्ये रत्नमयः सर्वतो वृत्तः ॥ ११ ॥

11. The Meru (mountain) is exactly one *yojana* (in height). It is light-producing, surrounded by the Himavat mountain, situated in the middle of the Nandana forest, made of jewels, and cylindrical in shape.

The height of the Meru mountain taught here is quite different from the teachings of the *Purāṇas*. It is also different from the teachings of the Buddhists and the Jainas.

According to the *Purāṇas*, the Meru mountain is 84,000 *yojanas* high, of which 16,000 *yojanas* lie inside the Earth.[2] According to the Buddhists, it is 1,60,000 *yojanas* high, of which 80,000 *yojanas* lie submerged in water and 80,000 *yojanas* above the Earth.[3] According to the Jainas, it is 1,00,000 *yojanas* high, of which 1000 *yojanas* lie inside the Earth and 99,000 *yojanas* outside the Earth.[4]

The commentator Nīlakaṇṭha thinks that the above stanza is meant to refute the enormous size of the Meru advocated in the *Purāṇas* and elsewhere. The commentators Bhāskara I, Someśvara and Raghunātharāja, however, reconcile the two views by interpreting the word Meru as meaning "the highest peak of the Meru mountain".

It seems that, according to the instruction of verse 8 above, the maximum uplift of the earth cannot exceed one *yojana* and so the height of any mountain cannot be greater than one *yojana*. This is perhaps the reason that Āryabhaṭa takes the height of the Meru mountain as one *yojana* only and not more.

Combining the instructions given in *Ā*, i. 7 with those given above, we see that, according to Āryabhaṭa I, the Meru mountain is cylindrical in shape, with its diameter and height each equal to one *yojana*.

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1. D. E. मात्रं

2. See *Vāyu-purāṇa*, ch. 34, *gāthā* 1-45 ; ch. 35, *gāthā* 11-32 ; *Viṣṇupurāṇa*, *Aṁśa* 2, ch. 2, *gāthā* 5-19 ; *Mārkaṇḍeya-purāṇa*, ch. 54, *gāthā* 5-19 ; *Matsya-purāṇa*, ch. 113, *gāthā* 4-40.

3. See *Abhidharmakoṣa* of Vasubandhu.

4. See *Lokaprakāśa*, 18.15-16.

In calling the Meru mountain as 'light-producing' Āryabhaṭa I probably has in mind the 'northern lights' or the 'aurora', which Robert H. Baker describes in the following words :

> "Characteristic of many displays of the 'northern lights' of our atmosphere is a luminous arch across the northern sky, having its apex in the direction of the geomagnetic pole. Rays like searchlight beams reach upward from the arch, while bright draperies may spread to other parts of the sky, altogether often increasing its brightness from 10 to 100 times that of the ordinary night sky. ....
>
> "The light of the aurora is believed to be produced by the streams of protons and electrons, which emerge from solar upheavals and are trapped by the Earth's magnetic field ......
>
> "Most of the light of an auroral display is produced in the colors green, red and blue by the combining of electrons with oxygen atoms and nitrogen molecules...."[1]

The Meru mountain is supposed to be made up of jewels of different colours because the light of an auroral display is of various colours.

THE MERU AND THE BAḌAVĀMUKHA

स्वर्मेरू स्थलमध्ये नरको बडवामुखं[2] च जलमध्ये ।
अमरमरा[3] मन्यन्ते परस्परमधःस्थितान्[4] नियतम् ॥ १२ ॥

12. **The heaven and the Meru mountain are at the centre of the land (*i.e.*, at the north pole); the hell and the Baḍavāmukha are at the centre of the water (*i e.*, at the south pole).[5] The gods (residing at the Meru mountain) and the demons (residing at the Baḍavā-**

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1. See Robert H. Baker, *Astronomy*, East-West Student Edition, New Delhi, 1965, pp. 312-14.

2. E. Go. Pa. मुखः; F. मुखं rev. to मुखः । Pṛ reads स्वर्मेरुः स्थल-मध्ये तदधो वडवामुखं

3. F. असुरसुरा ; Pṛ. अमरासुरा

4. Nī. So. Ya. स्थिता

5. *Cf.* *ŚiDVṛ*, II, iv, 4 ; *VSi*, Gola, vii. 11.

mukha) consider themselves positively and permanently below each other.[1]

The above statement is based on the conception that half of the Earth lying north of the equator is land and half of the Earth lying south of the equator is water.

THE FOUR CARDINAL CITIES

उदयो यो लङ्कायां सोऽस्तमयः[2] सवितुरेव सिद्धपुरे ।
मध्याह्नो यवकोट्यां[3] रोमक[4]विषयेऽर्धरात्रं[5] स्यात् ॥ १३ ॥

13. **When it is sunrise at Laṅkā, it is sunset at Siddhapura, midday at Yavakoṭi, and midnight at Romaka.**[6]

The time-distance relation is explained here with the help of four cities supposed to lie on the equator separated by one-quarter of the Earth's circumference.

Laṅkā is supposed to be at the place where the meridian of Ujjayinī (long. 75°.43 E., lat. 23°.09 N) intersects the equator, Yavakoṭi 90° to the east of Laṅkā, Romaka 90° to the west of Laṅkā, and Siddhapura diametrically opposite to Laṅkā.

POSITIONS OF LAṄKĀ AND UJJAYINĪ

स्थलजलमध्याल्लङ्का भूकक्ष्याया भवेच्चतुर्भागे ।
उज्जयिनी लङ्कायाः तच्चतुरंशे[7] समोत्तरतः ॥ १४ ॥

14. **From the centres of the land and the water, at a distance of one-quarter of the Earth's circumference, lies Laṅkā; and from Laṅkā, at a distance of one-fourth thereof, exactly northwards, lies Ujjayini.**[8]

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1. *Cf. PSi*, xiii. 3 ; *MSi*, xvi. 7 (a-b).
2. F. लङ्कायामस्तमयः
3. So. यमकोट्यां
4. D. लोमक
5. B. विषये निशीथं ; Nī. So. विषयेऽर्धरात्रः
6. *Cf. PSi*, xv. 23.
7. Nī. Sū. पञ्चदशांशे; Gh. Nī. Pa. Ra. Sū. note both readings
8. The same statement is made in *KR*, i. 33 (a-b); *ŚiDVṛ*, II, *Bhuvanakoṣa*, 40 (c-d) ; *SiŚi*, II, *Bhuvanakoṣa*, 15 (a-b).

The positions of Laṅkā and Ujjayinī have been given because the Hindu prime meridian is supposed to pass through them. By stating the positions of Laṅkā and Ujjayinī, Āryabhaṭa has, by implication, defined the position of the prime meridian.

The distance of Ujjayinī from Laṅkā as stated in the above passage is one-sixteenth of the Earth's circumference. This makes the latitude of Ujjayinī equal to 22° 30′ N. This is in agreement with the teachings of the earlier followers of Āryabhaṭa, such as Bhāskara I[1] (A. D. 629), Deva[2] (A. D. 689), and Lalla[3] and the interpretations of the commentators Someśvara, Sūryadeva (b. A.D. 1191) and Parameśvara (A.D. 1431).[4] Even the celebrated Bhāskara II[5] (A.D. 1150) has chosen to adopt it.

But Brahmagupta (A.D. 628) differed from this view. He takes Ujjayinī at a distance of one-fifteenth of the Earth's circumference from Laṇkā[6], and likewise the latitude of Ujjayinī as equal to 24° N. Some of the commentators of *Āryabhaṭīya*, who favoured Brahmagupta's view, changed the reading *taccaturaṁśe* into *pañcadaśaṁśe.* The commentator Sūryadeva, who first interprets the original reading *taccaturaṁśe*, later remarks :

Ujjayinī laṅkāyāḥ pañcadaśaṁśe samottarataḥ |

(*i. e.*, Ujjayinī is at a distance of one-fifteenth of the Earth's circumference to the exact north of Laṅkā) is the proper reading because Brahmagupta writes :

Laṅkottarato 'vantī bhūparidheḥ pañcadaśabhāge |

(*i. e.*, Avantī is to the north of Laṅkā at a distance of one-fifteenth of the Earth's circumference)."

In defence of the reading *pañcadaśaṁśe*, Sūryadeva again says :

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1. See his comm, on *Ā*, i. 7, where he gives the distance between Laṅka and Ujjayinī as approximately equal to 200 *yojanas*.
2. See *KR*, i. 33 (a-b).
3. See *ŚiDVṛ*, II, *Bhuvanakoṣa*, 40 (c-d).
4. Parameśvara notes the other reading पञ्चदशांशे also.
5. See *SiŚi*, II, *Bhuvanakoṣa*, 15 (a-b).
6. See *BrSpSi*, xxi. 9 (c-d).

"24° to the north of Laṅkā lies Ujjayinī. So, when the Sun is situated at the end of Gemini, then, due to its greatest declination of 24°, it causes midday when it is exactly overhead at Ujjayinī. In a place to the north of Ujjayinī, the Sun is never exactly overhead. To the south (of Ujjayinī), it is exactly overhead when the Sun's north declination becomes equal to the latitude of the place. Thereafter it gets depressed towards the north. So the instruction of Ujjayinī for the knowledge of a place having a latitude equal to the Sun's greatest declination is appropriate. We do not see any use in the instruction of Ujjayinī lying at a distance of one-sixteenth of the Earth's circumference (to the north of Laṅkā), for its latitude being 22°30′ N., it is of no use anywhere...... So we have rightly said : *Ujjayinī laṅkāyāḥ pañcadaśāṁśe samottarataḥ.*"

The commentator Nīlakaṇṭha (1500 A.D.) mentions the reading *taccaturaṁśe* but adopts the reading *pañcadaśāṁśe* taking it to be correct. Writes he :

> "Some read *taccaturaṁśe*. According to them the word *tat* means one-fourth of the Earth's circumference, one-fourth of one-fourth is indeed one-sixteenth. So there is difference of meaning between the two. (However,) between facts there can be no option. So only one of the two readings is correct. Which of the two is correct can be decided upon from the equinoctial midday shadow at Ujjayinī. That the *janapada* of Ujjayinī lies at a distance of one-fifteenth of the Earth's circumference is well known from other works on astronomy. For the son of Jiṣṇu (*i.e.*, Brahmagupta) writes :
>
> > 'Avantī is to the north of Laṅkā at a distance of one-fifteenth of the Earth's circumference'.
>
> So also writes Varāhamihira, who belonged to Avantī :
>
> > 'When the Sun is at the end of Gemini, it revolves 24° above the horizon of the gods ; and at Avantī it is then exactly overhead (at midday)'.
>
> This shows that the latitude there is 24°. Now 24° is one-fifteenth of a circle and not one-sixteenth, because there are 360° in the whole circle and 24° is one-fifteenth of 360°. So the reading *pañcadaśāṁśe* is the correct reading."

But he adds :

> "However, that *janapada* being large and the latitude being different at different places, somewhere (in that *janapada*) a latitude of 24° is also possible. Whether it occurs at Ujjayinī or not, can be decided (only) by the people there. Varāhamihira has shown it to be 24° in respect of his village. Following him the son of Jiṣṇu, too, has said the same. But Ujjayinī is to the south of that (village). There a latitude of 22½ degrees is also possible. In that case the other reading (*taccaturaṁśe*) would be correct, for latitude has been stated (here) for Ujjayinī (and not for the village of Varāhamihira)."

The commentator Raghunātha-rāja (1597 A.D.) adopts the reading *pañcadaśāṁśe*. He interprets the reading *taccaturaṁśe* also, but he prefers the other reading on the same grounds as given by Sūryadeva.

The majority of the Hindu astronomers, however, favours Brahmagupta's view and takes the latitude of Ujjayinī as 24° N. But there is no doubt that according to Āryabhaṭa I it is 22° 30′ N.

VISIBLE AND INVISIBLE PORTION OF THE BHAGOLA

भूव्यासार्धेनोनं दृश्यं देशात् समाद् भगोलार्धम् ।
अर्धं भूमिच्छन्नं भूव्यासार्धाधिकं चैव ॥ १५ ॥

15. **One half of the Bhagola as diminished by the Earth's semi-diameter is visible from a level place (free from any obstructions). The other one-half as increased by the Earth's semi-diameter remains hidden by the Earth.**[1]

What is meant is that that portion of the *Bhagola* is visible at a place O on the Earth's surface which lies above the sensible horizon at O, *i.e.*, which lies above the tangent plane to the Earth's surface at O, and that portion of the *Bhagola* which lies below the sensible horizon at O is invisible at O.

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1. *Cf. ŚiDVṛ* II, vi. 35.

From this we easily deduce that according to Āryabhaṭa I

Sun's mean horizontal parallax = 3′ 56″

Moon's mean horizontal parallax = 52′ 30″,

the corresponding modern values being 8″·794 and 57′ 2″·7, respectively.

MOTION OF THE BHAGOLA FROM THE NORTH AND SOUTH POLES

देवाः पश्यन्ति भगोलार्धमुदङ्मेरुसंस्थिताः सव्यम् ।
अर्धं त्वपसव्यगतं[1] दक्षिणबडवामुखे प्रेताः ॥ १६ ॥

16. **The gods living in the north at the Meru mountain (*i.e.*, at the north pole) see one half of the Bhagola as revolving from left to right (or clockwise); the demons living in the south at the Baḍavāmukha (*i.e.*, at the south pole), on the other hand, see the other half as revolving from right to left (or anti. clokwise).[2]**

VISIBILITY OF THE SUN TO THE GODS, MANES AND MEN

रविवर्षार्धं देवाः पश्यन्त्युदितं रविं तथा प्रेताः ।
शशिमासार्धं पितरः शशिगाः कुदिनार्धमिह मनुजाः ॥ १७ ॥

17. **The gods see the Sun, after it has risen, for half a solar year; so is done by the demons too.[3] The manes living on (the other side of) the Moon see the Sun for half a lunar month;[4] the men here see it for half a civil day.[5]**

This verse stating how long do the gods (living at the north pole), the demons (living at the south pole), the manes (living on the other side of the Moon) and men see the Sun after it has once risen.

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1. All others except So. अपसव्यगं तथार्धं
2. *Cf. PSi*, xiii. 9.
3. *Cf. PSi*, xiii. 27.
4. *Cf. PSi*, xiii. 38.
5. *Cf. PSi*, xv. 14.

2. *Khagola*

THE PRIME VERTICAL, MERIDIAN AND HORIZON

पूर्वापरमधऊर्ध्वं मण्डलमथ दक्षिणोत्तरं चैव ।
क्षितिजं समपार्श्वस्थं भानां यत्रोदयास्तमयौ ॥ १८ ॥

18. **The vertical circle which passes through the east and west points is the prime vertical, and the vertical circle passing through the north and south points is the meridian. The circle which goes by the side of the above circles (like a girdle) and on which the stars rise and set is the horizon.**[1]

P.C. Sengupta's remark that "here we have the rational horizon and not the apparent horizon" is incorrect.

Since the centre of the *Khagola* is at the observer lying on the surface of the Earth, the horizon is evidently the apparent or sensible horizon and not the rational horizon.

EQUATORIAL HORIZON

पूर्वापरदिग्लग्नं क्षितिजादक्षाग्रयोश्च लग्नं यत् ।
उन्मण्डलं भवेत्तत् क्षयवृद्धी यत्र दिवसनिशोः ॥ १९ ॥

19. **The circle which passes through the east and west points and meets (the meridian above the north point and below the south point) at distances equal to the latitude (of the place) from the horizon is the equatorial horizon (or six o' clock circle) on which the decrease and increase of the day and night are measured.**[2]

THE OBSERVER IN THE KHAGOLA

पूर्वापरदिग्रेखाधश्चोर्ध्वा[3] दक्षिणोत्तरस्था च ।
एतासां सम्पातो द्रष्टा यस्मिन् भवेद् देशे ॥ २० ॥

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1. *Cf. ŚiDVṛ*, II, ii. 2 ; *VSi, Gola*, iv 2. ; *SiŚe*, xvi. 29 (d) ; *SiŚi*, II, vi. 3 (c-d) ; vii. 2 (c-d).

2. *Cf. BrSpSi*, xxi. 50 ; *ŚiDVṛ*. II, ii. 3 ; *VSi, Gola*, iv. 3 ; *SiŚe*, xvi. 30 ; *SiŚi*, II, vi. 4 ; *SuSi*, II, iv, 4.

3. F. ध्वा, rev. to ध्वं

20. The east-west line, the nadir-zenith line, and the north-south line intersect where the observer is.[1]

What Āryabhaṭa I means to say is that the centre of the *Khagola* lies at the observer, or that (the position of) the observer forms the centre of the *Khagola.*

THE OBSERVER'S *DṚṄMAṆḌALA* AND *DṚKKṢEPAVṚTTA*

ऊर्ध्वमधस्ताद् द्रष्टुर्ज्ञेयं दृङ्मण्डलं ग्रहाभिमुखम् ।
दृक्क्षेपमण्डलमपि प्राग्लग्नं स्यात् त्रिराश्यूनम् ॥ २१ ॥

21. The great circle which is vertical in relation to the observer and passes through the planet is the *dṛṅmaṇḍala* (*i. e.*, the vertical circle through the planet). The vertical circle which passes through that point of the ecliptic which is three signs behind the rising point of the ecliptic is the *dṛkkṣepavṛtta*.

THE AUTOMATIC SPHERE (*GOLA-YANTRA*)

काष्ठमयं समवृत्तं समन्ततः समगुरुं लघुं गोलम् ।
पारद्[2]तैलजलैस्तं भ्रमयेत् स्वधिया च कालसमम् ॥ २२ ॥

22. The Sphere (*Gola-yantra*) which is made of wood, perfectly spherical, uniformly dense all round but light (in weight) should be made to rotate keeping pace with time with the help of mercury, oil and water by the application of one's own intellect.

The *Gola-yantra* is the representation of the *Bhagola.*

The method used by Āryabhaṭa for rotating the Sphere (*Bhagola*) at the rate of one rotation per twentyfour hours may be briefly described in the words of the commentator Sūryadeva as follows :

> "Having set up two pillars on the ground, one towards the south and the other towards the north, mount on them the

1. *Cf. ŚiDVṛ*, II, vi. 33-34.

2. E. Nī. Pa. So. पारत

ends of the iron needle (rod) (which forms the axis of rotation of the Sphere). In the holes of the Sphere, at the south and north poles, pour some oil, so that the sphere may rotate smoothly. Then, underneath the west point of the Sphere, dig a pit and put into it a cylindrical jar with a hole in the bottom and as deep as the circumference of the Sphere. Fill it with water. Then having fixed a nail at the west point of the Sphere, and having fastened one end of a string to it, carry the string downwards along the equator towards the east point, then stretch it upwards and carry it to the west point (again), and then fasten to it a dry hollow gourd (appropriately) filled with mercury and place it on the surface of water inside the cylindrical jar underneath, which is already filled with water. Then open the hole at the bottom of the jar so that with the outflow of water, the water inside the jar goes down. Consequently, the gourd which, due to the weight of mercury within it, does not leave the water, pulls the Sphere westwards. The outflow of water should be manipulated in such a way that in 30 *ghaṭīs* (=12 hours) half the water of the jar flows out and the Sphere makes one-half of a rotation, and similarly, in the next 30 *ghaṭīs* the entire water of the jar flows out, the gourd reaches the bottom of the jar and the Sphere performs one complete rotation. This is how one should, by using one's intellect, rotate the Sphere keeping pace with time."

3. *Spherical Astronomy*

(1) Diurnal motion

THE LATITUDE-TRIANGLE

दृग्गोलार्धकपाले ज्यार्धेन विकल्पयेद् भगोलार्धम् ।
विषुवज्जीवाक्षभुजा तस्यास्त्ववलम्बकः[1] कोटिः ॥ २३ ॥

23. **Divide half of the Bhagola lying in the visible half of the Khagola by means of Rsines (so as to form latitude-triangles). The Rsine of the latitude is the base of a latitude-triangle. The Rsine of the colatitude is the upright of the same (triangle).**

The statement "half of the *Bhagola* lying in the visible half of the *Khagola*," implies that the radius of the Earth is disregarded

1. A. तस्या ह्यवलम्बकः

here and the centre of the *Khagola* is supposed to be coincident with the centre of the *Bhagola*. What is meant is the standard *Khagola*, i.e., *Khagola* for the centre of the Earth.

A right-angled plane triangle whose sides are proportional to Rsin θ, Rcos θ and R, where R (=3438′) is the radius of the *Bhagola*, is called a latitude-triangle (*akṣa-kṣetra*). The right-angled plane triangle whose sides are equal to Rsin θ, Rcos θ and R is the main latitude-triangle, defined above.

The latitude-triangles play an important role in the solution of the spherical triangles in Indian astronomy. For, a number of results in astronomy are obtained simply by comparing two latitude-triangles. Because of this importance of the latitude-triangles, Āryabhaṭa II (c. 950 A. D.) and Bhāskara II (1150 A.D.) have given a list of such triangles in their works. "It is only he who is versed in the latitude-triangles," adds Bhāskara II, "that enjoys respect, fortune, fame, and happiness".[1]

The latitude-triangles

(Āryabhaṭa II's list)

	Base	Upright	Hypotenuse
(1)	Rsin θ	Rcos θ	R
(2)	equinoctial midday shadow	gnomon (=12)	hypotenuse of equinoctial midday shadow
(3)	earthsine	Rsin δ	*agrā*
(4)	*unmaṇḍalaśaṅku*	first part of *agrā*	Rsin δ
(5)	other part of *agrā*	*unmaṇḍalaśaṅku*	earthsine
(6)	*agrā*	*samaśaṅku*	*taddhṛti*
(7)	Rsin δ	upper part of *taddhṛti* (*taddhṛti*—earthsine)	*samaśaṅku*
	(Bhāskara II's additional triangle)		
(8)	first part of *agrā*	upper part of *samaśaṅku*	upper part of *taddhṛti*

1. क्षेत्राणि यान्यक्षभवानि तेषां विद्येव मानार्थयशःसुखानाम् ॥
SiŚi, Grahagaṇita, iii. 13 (c-d)

Explanation : When a heavenly body is on the six o' clock circle, the perpendicular dropped from it on the plane of the horizon is called *unmaṇḍalaśaṅku* ; the distance of the foot of the perpendicular from the east-west line is called the first part of *agrā* ; the distance of the heavenly body from the rising-setting line is called the earthsine. When the heavenly body is on the prime vertical, the perpendicular dropped from it on the east-west line is called *sama-śaṅku* ; the perpendicular dropped from it on the rising-setting line is called *taddhṛti* ; the distance between the east-west line and the rising-setting line is called *agrā*. When a perpendicular is dropped from the foot of the *samaśaṅku* on the *taddhṛti*, the latter is divided into two parts called upper and lower ; when a perpendicular is dropped from the foot of this perpendicular on the *samaśaṅku*, the latter is divided into two parts called upper and lower; when from the same foot a perpendicular is dropped on the *agrā*, the latter is divided into two parts called the 'first part of *agrā*' and the 'other part of *agrā*'.

RADIUS OF THE DAY-CIRCLE

इष्टापक्रमवर्गं व्यासार्धकृतेर्विशोध्य यन्मूलम्[1] ।
विषुवदुदग्दक्षिणतस्तदहोरात्रार्धविष्कम्भः ॥ २४ ॥

24. **Subtract the square of the Rsine of the given declination from the square of the radius, and take the square root of the difference. The result is the radius of the day circle, whether the heavenly body is towards the north or towards the south of the equator.**[2]

That is,

$$\text{day radius} = \sqrt{R^2 - (\text{Rsin } \delta)^2}, \qquad (1)$$

$$\text{Rsin } \delta = \frac{\text{Rsin } \lambda \times \text{Rsin } 24^\circ}{R}, \qquad (2)$$

λ and δ being, respectively, the Sun's tropical longitude and declination of the heavenly body.

Āryabhaṭa does not state formula (2) for finding Rsin δ, because it can be easily derived by applying the rule of three as follows : "When the Rsine of the Sun's tropical longitude is equal to R, the Rsine of the Sun's declination is equal to Rsin 24°; what then

1. B. C. E. Sū. मूलं यत्
2. *Cf. MBh*, iii. 6.

will be the value of the Rsine of the Sun's declination when the Rsine of the Sun's tropical longitude has the value Rsin λ ? The result is Rsin δ."

RIGHT ASCENSIONS OF ARIES, TAURUS AND GEMINI

इष्टज्यागुणितमहोरात्रव्यासार्धमेव काष्ठान्त्यम् ।
स्वाहोरात्रार्धहृतं[1] [2]फलमजाल्लङ्कोदयप्राग्ज्या[3] ॥ २५ ॥

25. Multiply the day radius corresponding to the greatest declination (on the ecliptic) by the desired Rsine (of one, two or three signs) and divide by the corresponding day radius : the result is the Rsine of the right ascension (of one, two or three signs), measured from the first point of Aries along the equator.[4]

Let α, β and γ denote the right ascensions of one sign, two signs and three signs, respectively, and δ_1, δ_2 and δ_3 the declinations at the last points of the signs Aries, Taurus and Gemini, respectively. Then

$$\text{Rsin } \alpha = \frac{\text{Rsin } 30^\circ \times \text{Rcos } 24^\circ}{\text{Rcos } \delta_1} \tag{1}$$

$$\text{Rsin } \beta = \frac{\text{Rsin } 60^\circ \times \text{Rcos } 24^\circ}{\text{Rcos } \delta_2} \tag{2}$$

$$\text{Rsin } \gamma = \frac{\text{Rsin } 90^\circ \times \text{Rcos } 24^\circ}{\text{Rcos } \delta_3} \tag{3}$$

Now, Rcos $\delta_1 = 3366'$, Rcos $\delta_2 = 3218'$ and Rcos $\delta_3 = 3141'$. Hence substituting these values and simplifying, we get $\alpha = 1670'$ $\beta = 3465'$ and $\gamma = 5400'$. Consequently,

right ascension of Aries $= \alpha = 1670$ respirations

right ascension of Taurus $= \beta - \alpha = 1795$ respirations

right ascension of Gemini $= \gamma - \beta = 1935$ respirations.

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1. So. हृतं
2. Ra. मेषात् for फलमजात्
3. Ra. So. प्राग्ज्या:
4. *Cf. MBh*, iii. 9.

The right ascensions of Aries, Taurus and Gemini in the reverse order are the right ascensions of Cancer, Leo and Virgo ; and the right ascensions of the first six signs, Aries etc. in the reverse order are the right ascensions of the last six signs, Libra etc.

Table 22. Right ascensions of the signs of the ecliptic

Sign	Right ascension in respirations	Sign
1 Aries	1670	12 Pisces
2 Taurus	1795	11 Aquarius
3 Gemini	1935	10 Capricorn
4 Cancer	1935	9 Sagittarius
5 Leo	1795	8 Scorpio
6 Virgo	1670	7 Libra

The Indian method for deriving formula (1) is as follows :

Consider the Celestial Sphere for a place on the equator. Let the first point of Aries coincide with the east point of the horizon ; and let A be the last point of the sign Aries, AB the perpendicular from A on the eastwest line, and AC the perpendicular from A on the plane of the horizon. Also let G be the last point of the sign Gemini, GO the perpendicular from G on the east-west line and GM the perpendicular from G on the plane of the horizon.

Then comparing the triangles ABC and GOM, which are evidently similar, we have

$$AC = \frac{AB \times GM}{GO}$$

$$= \frac{R\sin 30° \times R\cos 24°}{R}$$

Now $R\sin\alpha : R :: AC : R\cos\delta$. Therefore

$$R\sin\alpha = \frac{AC \times R}{R\cos\delta} = \frac{R\sin 30° \times R\cos 24°}{R\cos\delta}$$

The rationales of formulae (2) and (3) are similar.

EARTHSINE

इष्टापक्रमगुणितामक्षज्यां लम्बकेन हृत्वा[1] या ।
स्वाहोरात्रे क्षितिजा क्षयवृद्धिज्या दिननिशोः सा ॥ २६ ॥

26. **The Rsine of latitude multiplied by the Rsine of the given declination and divided by the Rsine of colatitude gives the earthsine, lying in the plane of the day circle. This is also equal to the Rsine of half the excess or defect of the day or night (in the plane of the day circle).[2]**

That is,

$$\text{earthsine} = \frac{\text{Rsin } \delta \times \text{Rsin } \phi}{\text{Rcos } \phi}$$

This result may be easily obtained by comparing the following latitude-triangles ;

	Base	Upright	Hypotenuse
(1)	earthsine	Rsin δ	*agrā*
(2)	Rsin ϕ	Rcos ϕ	R

By the 'excess or defect of the day or night' is meant the amount by which the day or night at the local place is greater or less than 30 *ghaṭīs* (or 12 hours).

The earthsine, as the text says, is the Rsine of half the excess or defect of the day or night in the plane of the day circle. Since the time is measured on the equator, one should first find the corresponding Rsine in the plane of the equator and then reduce that to the arc of the equator.

The Rsine of half the excess or defect of the day or night in the plane of the equator is called *carārdhajyā* and is obtained by the following formula :

$$\textit{carārdhajyā} = \frac{\text{earthsine} \times \text{R}}{\text{day radius}}$$

The corresponding arc of the equator is called *carārdha* and gives the amount by which the semi-duration of the day or night at the local place is greater or less than 15 *ghaṭīs*.

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1. F. भक्त्वा for हृत्वा
2. *Cf. PSi*, iv. 34 ; *MBh*, iii. 6.

The *carārdha* is also equal to the difference between the oblique and right ascensions and so it is called the 'ascensional difference'. The oblique ascension is the time of rising of an arc of the ecliptic at the local place and the right ascension is the time of rising of an arc of the ecliptic at the equator.

RISING OF THE FOUR QUADRANTS AND OF THE INDIVIDUAL SIGNS

उद्यति हि चक्रपादश्चरदलहीनेन दिवसपादेन ।
प्रथमोऽन्त्यश्चाथान्यौ[1] तत्सहितेन[2] क्रमोत्क्रमशः[3] ॥ २७ ॥

27. **The first as well as the last quadrant of the ecliptic rises (above the local horizon) in one quarter of a sidereal day diminished by (the *ghaṭīs* of) the ascensional difference. The other two (*viz.* the second and third quadrants) rise in one quarter of a sidereal day as increased by the same (*i.e.* the *ghaṭīs* of the ascensional difference). The times of rising of the individual signs (Aries, Taurus and Gemini) in the first quadrant are obtained by subtracting their ascensional differences from their right ascensions in the serial order ; in the second quadrant by adding the ascensional differences of the same signs to the corresponding right ascensions in the reverse order. The times of risings of the six signs in the first and second quadrants (Aries, etc.) taken in the reverse order give the risings of the six signs in the third and fourth quadrants (Libra, etc.).[4]**

Let Fig. 10 represent the Celestial Sphere (*Khagola*) for the local place. SEN is the horizon, RET the equator, UEV the ecliptic and PEQ the equatorial horizon. The small circle WBV is the day circle through V (the end of the first quadrant of the ecliptic).

EV is the first quadrant of the ecliptic. At the moment the first point of Aries coincides with E. With the motion of the Celestial Sphere E will move along the equator and V along the diurnal circle

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1. D. श्च तथान्यो
2. E. सहितौ
3. A. E. Gh. Go. Nī. Pa. क्रमोत्क्रमतः
4. *Cf. PSi*, iv. 31 ; *LBh*, iii. 6.

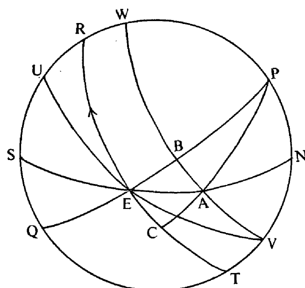


Fig. 10

in the direction of the arrowhead. When V reaches A, the whole of the first quadrant of the ecliptic, which is at the moment on the point of rising above the horizon, will be above the local horizon. So the time of rising of the first quadrant of the ecliptic at the local place is the time taken by V in moving from V to A, or, what is the same thing, the time taken by T in moving from T to C. This time is given by the arc

$$TC \text{ or } ET-EC$$

of the equator (because time is measured on the equator). Since ET is one-fourth of the equator, it corresponds to one-quarter of a sidereal day. So the time of rising of the first quadrant of the ecliptic

=one quarter of a sidereal day—*ghaṭīs* corresponding to arc EC

=15 *ghaṭīs*—*ghaṭīs* of ascensional difference. (1)

Also, the first quadrant of the ecliptic is, at the moment, at the point of rising above the equatorial horizon QEP. When the point V reaches the point B, the first quadrant of the ecliptic will be completely above the equatorial horizon. The time taken by V to reach B is equal to the time taken by T in reaching E. So the time of rising of the first quadrant of the ecliptic above the equatorial horizon is equal to the arc ET of the equator, which has been just shown to correspond to one quarter of a sidereal day or 15 *ghaṭīs*. This differs from (1) by the time given by the arc EC of the equator. EC therefore gives the difference between the times of rising of the first quadrant at the local and equatorial places. EC is, therefore, called the 'ascensional difference' of the first quadrant (or the ascensional difference of the last point of the first quadrant).

Hence, from (1), we have

(1) Time of rising of the first quadrant at the local place

= 15 *ghaṭīs—ghaṭīs* of the ascensional difference.

When the first point of Aries is at E, the first point of Libra is at the west point W. The first point of Libra will reach the point E exactly after 30 *ghaṭīs* and then the second quadrant of the ecliptic will be completely above the local horizon. Hence we have

(2) Time of rising of the second quadrant of the ecliptic at the local place

= 30 *ghaṭīs* – (15 *ghaṭīs—ghaṭīs* of asc. diff.)

= 15 *ghaṭīs*+*ghaṭīs* of asc. diff.

Similarly, we can show that

(3) Time of rising of the third quadrant of the ecliptic at the local place = 15 *ghaṭīs* + *ghaṭīs* of asc. diff.

(4) Time of rising of the fourth quadrant of the ecliptic at the local place = 15 *ghaṭīs* – *ghaṭīs* of asc. diff.

Proceeding exactly in the same manner, we can show that

Time of rising of the sign Aries at the local place

= Time of rising of the sign Aries at the equator—asc. diff. of the last point of Aries. (2)

Time of rising of the signs Aries and Taurus at the local place

= Time of rising of the signs Aries and Taurus at the equator—asc. diff. of the last point of Taurus. (3)

Time of rising of the signs Aries, Taurus and Gemini at the local place

= Time of rising of the signs Aries, Taurus and Gemini at the equator—asc. diff. of the last point of Gemini. (4)

Diminishing (2), (3), (4), each by the preceding (if any), we have

Time of rising of the sign Aries at the local place

= Time of rising of the sign Aries at the equator—asc. diff. of Aries.

Time of rising of the sign Taurus at the local place

= Time of rising of the sign Taurus at the equator – (asc. diff. of the last point of Taurus—asc. diff. of the last point of Aries)

= Time of rising of the sign Taurus at the equator—asc. diff. of Taurus.

Time of rising of the sign Gemini at the local place

=Time of rising of the sign Gemini at the equator−(asc. diff. of the last point of Gemini−asc. diff. of the last point of Taurus)

=Time of rising of the sign Gemini at the equator−asc. diff. of Gemini.

Let A, B, C be the times of rising of the signs Aries, Taurus, and Gemini at the equator and a, b, c the ascensional differences of the same signs in their respective order. Then the times of rising of the signs at the local place are as shown in the following table.

Table 23. Times of rising of the signs at the local place

Sign	Time of rising	Sign
1 Aries	$A-a$	12 Pisces
2 Taurus	$B-b$	11 Aquarius
3 Gemini	$C-c$	10 Capricorn
4 Cancer	$C+c$	9 Sagittarius
5 Leo	$B+b$	8 Scorpio
6 Virgo	$A+a$	7 Libra

RSINE OF THE ALTITUDE

स्वाहोरात्रेष्टज्यां क्षितिजादवलम्बकाहतां कृत्वा ।
विष्कम्भार्धविभक्ते दिनस्य गतशेषयोः शङ्कुः ॥ २८ ॥

28. Find the Rsine of the arc of the day circle from the horizon (up to the point occupied by the heavenly body) at the given time ; multiply that by the Rsine of the colatitude and divide by the radius : the result is the Rsine of the altitude (of the heavenly body) at the given time elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon.

By 'the Rsine of the arc of the day circle from the horizon up to the point occupied by a heavenly body', is meant the distance of the heavenly body from the rising-setting line, which is known as *iṣṭahṛti*.

Thus the formula in the text may be stated as

$$\text{Rsin (Sun's altitude)} = \frac{\textit{iṣṭahṛti} \times \text{Rcos } \phi}{\text{R}}$$

This formula may be obtained by comparing the following latitude-triangles :

	Base	Upright	Hypotenuse
(1)	*śaṅkvagra* or *śaṅkutala*	Rsin (Sun's altitude)	*iṣṭahṛti*
(2)	Rsin ϕ	Rcos ϕ	R

The method intended by Āyabhaṭa I may be fully explained in the case of the Sun as follows :

> "With the help of the Sun's declination and the local latitude calculate the Sun's ascensional difference. Subtract the Sun's ascensional difference from or add that to the given time reduced to *asus* (1 *ghaṭī*=360 *asus*), according as the Sun is in the northern or southern hemisphere. By the Rsine of that difference or sum multiply the day radius and divide by the radius. If the Sun is in the northern hemisphere, add the earthsine to the result obtained ; if the Sun is in the southern hemisphere, subtract the earthsine from the result obtained : the result is the *iṣṭahṛti*. Multiply that by the Rsine of the colatitude and divide by the radius : the result is the Rsine of the Sun's altitude."
>
> 'When the Sun is the northern hemisphere and the given time reduced to *asus* is less than the Sun's ascensional difference reduced to minutes of arc, one should proceed as follows :
>
> 'Subtract the *asus* of the given time from the minutes of the Sun's ascensional difference ; multiply the difference by the day radius and divide by the radius. Subtract whatever is obtained from the earthsine : the result is the *iṣṭahṛti*. Multiply that by the Rsine of colatitude and divide by the radius ; the result is the Rsine of the Sun's altitude as before'."[1]

1. See *MBh*, iii. 18-20, 25. Also see *PSi*, iv. 41-43.

ŚAṄKVAGRA

विषुवज्जीवागुणितः[1] स्वेष्टः शङ्कुः स्वलम्बकेन हृतः ।
अस्तमयोदयसूत्राद् दक्षिणतः सूर्यशङ्क्वग्रम् ॥ २९ ॥

29. Multiply the Rsine of the Sun's altitude for the given time by the Rsine of latitude and divide by the Rsine of colatitude : the result is the Sun's *śaṅkvagra*, which is always to the south of the Sun's rising-setting line.[2]

The Sun's *śaṅkvagra* is the distance of the Sun's projection on the plane of the observer's horizon from the Sun's rising-setting line. Or, it is the projection of the *iṣṭahṛti* on the plane of the observer's horizon.

The formula stated in the text is

$$\text{Sun's } \textit{saṅkvagra} = \frac{\text{Rsin (Sun's altitude)} \times \text{R sin } \phi}{\text{Rcos } \phi},$$

which can be easily derived by comparing the following latitude-triangles :

	Base	Upright	Hypotenuse
(1)	Sun's *śaṅkvagra*	Rsine (Sun's altitude)	*iṣṭahṛti*
(2)	Rsin ϕ	Rcos ϕ	R .

Although the rule is stated for the Sun, it is applicable to any heavenly body whatsoever.

SUN'S AGRĀ

परमापक्रमजीवामिष्टज्यार्धाहतां ततो विभजेत् ।
ज्या लम्बकेन लब्धाऽर्काग्रा पूर्वापरे क्षितिजे ॥ ३० ॥

30. Multiply the Rsine of the (Sun's tropical) longitude for the given time by the Rsine of the Sun's greatest declination and then divide by the Rsine of colatitude : the resulting Rsine is the Sun's *agrā* on the eastern or western horizon.[3]

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1. Gh. निहृत: for गुणित:
2. *Cf. MBh*, iii. 54 ; *LBh*, iii. 16.
3. *Cf. MBh*, iii. 37 ; *LBh*, iii. 21.

The Sun's *agra* is the distance of the rising or setting Sun from the east-west line.

The formula stated in the text is

$$\text{Sun's } agra = \frac{\text{Rsin } \lambda \times \text{Rsin } 24^\circ}{\text{Rcos } \phi},$$

where λ is the Sun's tropical longitude, and ϕ the latitude of the place.

This formula may be obtained as follows :

Comparing the latitude-triangles :

	Base	Upright	Hypotenuse
(1)	earthsine	Rsin δ	*agra*
(2)	Rsin ϕ	Rcos ϕ	R

we get

$$\text{Sun's } agra = \frac{\text{Rsin } \delta \times \text{R}}{\text{Rcos } \phi}.$$

But

$$\text{Rsin } \delta = \frac{\text{Rsin } \lambda \times \text{Rsin } 24^\circ}{\text{R}}.$$

Therefore

$$\text{Sun's } agra = \frac{\text{Rsin } \lambda \times \text{Rsin } 24^\circ}{\text{R cos } \phi}.$$

RSINE OF THE SUN'S PRIME VERTICAL ALTITUDE

सा विषुवज्ज्योना चेद् विषुवदुदग्लम्बकेन सङ्गुणिता ।
विषुवज्ज्यया विभक्ता लब्धः पूर्वापरे शङ्कुः ॥ ३१ ॥

31. **When that (*agra*) is less than the Rsine of the latitude and the Sun is in the northern hemisphere, multiply that (Sun's *agra*) by the Rsine of colatitude and divide by the Rsine of latitude : the result is the Rsine of the Sun's altitude when the Sun is on the prime vertical.**[1]

---

1. *Cf. MBh*, iii. 37 (c-d)-38.

That is,

$$\text{Rsin } a = \frac{\text{Sun's } agra \times \text{Rcos } \phi}{\text{Rsin } \phi},$$

where a is the Sun's prime vertical altitude.

This formula may be easily derived by comparing the following latitude-triangles :

	Base	Upright	Hypotenuse
(1)	*agrā*	Rsin a	*iṣṭahṛti*
(2)	Rsin ϕ	Rcos ϕ	R

The conditions necessary for the existence of the prime vertical altitude of the Sun are : (1) that the Sun should be in the northern hemisphere, and (2) that the Sun's declination should be less than the latitude of the place. The condition given by Āryabhaṭa that the Sun's *agrā* should be less than the Rsine of the latitude is incorrect. Brahmagupta (A.D. 628) has therefore rightly criticised Āryabhaṭa on this account :

> "The statement (of Āryabhaṭa) that the Sun, in the northern hemisphere, enters the prime vertical when the (Sun's) *agrā* is less than the Rsine of the latitude is incorrect, because this happens when the Rsine of the (Sun's) declination satisfies this condition (and not the Sun's *agrā*)."[1]

It is interesting to note that the commentator Bhāskara I (A.D. 629), committed the same error in his *Mahā-Bhāskarīya*[2], but he has corrected himself in his *Laghu-Bhāskarīya*.[3]

Sūryadeva (b. A.D. 1191), Someśvara, and other commentators, however, have interpreted the word *sā* as referring to the Sun's declination and not to the Sun's *agrā*.

Although the rule in vss. 30-31 is stated for the Sun, it is applicable to any heavenly body whatsoever.

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1. *BrSpSi*, xi. 22.
2. *MBh*, iii. 37.
3. *LBh*, iii. 22.

SUN'S GREATEST GNOMON AND THE SHADOW THEREOF

क्षितिजादुन्नतभागानां या ज्या सा परो भवेच्छङ्कुः ।
मध्यान्नतभागज्या छाया शङ्कोस्तु[1] तस्यैव ॥ ३२ ॥

32. **The Rsine of the degrees of the (Sun's) altitude above the horizon (at midday when the Sun is on the meridian) is the greatest gnomon (on that day). The Rsine of the (Sun's) zenith distance (at that time) is the shadow of the same gnomon.**

The Sun's zenith distance at midday

$$=\phi \sim \delta \text{ or } \phi+\delta,$$

according as the Sun is in the northern or southern hemisphere.

Consequently, the greatest gnomon or the Rsine of the Sun's altitude at midday

$$=\text{Rcos } (\phi \sim \delta) \text{ or Rcos } (\phi+\delta),$$

and the shadow of the greatest gnomon or the Rsine of the Sun's zenith distance at midday

$$= \text{Rsin } (\phi \sim \delta) \text{ or Rsin } (\phi+\delta),$$

according as the Sun is in the northern or southern hemisphere.

(2) Parallax in a solar eclipse

RSINE OF THE ZENITH DISTANCE OF THE CENTRAL ECLIPTIC POINT

मध्यज्योदयजीवासंवर्गे व्यासदलहृते यत् स्यात् ।
तन्मध्यज्याकृत्योर्विशेषमूलं स्वदृक्क्षेपः ॥ ३३ ॥

33. **Divide the product of the *madhyajya* and the *udayajyā* by the radius. The square root of the difference between the squares of that (result) and the *madhyajya* is the (Sun's or Moon's) own *dṛkkṣepa*.**[2]

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1. F. शङ्कोश्च
2. *Cf. PSi,* ix. 19-20 ; *MBh,* v. 19.

The Sun's *madhyajyā* is the Rsine of the zenith distance of the meridian ecliptic point. The Sun's *udayajyā* is the Rsine of the amplitude of the rising point of the ecliptic. The Sun's *dṛkkṣepa*(*jyā*) is the Rsine of the zenith distance of that point of the ecliptic which is at the shortest distance from the zenith.

The Moon's *madhyajyā* is the Rsine of the zenith distance of that point of the Moon's orbit which lies on the observer's meridian. The Moon's *udayajyā* is the Rsine of the amplitude of that point of the Moon's orbit which lies on the eastern horizon of the observer. The Moon's *dṛkkṣepa*(*jyā*) is the Rsine of that point of the Moon's orbit which is at the shortest distance from the zenith.

Let Z be the zenith, M the meridian ecliptic point and C that point of the ecliptic which is at shortest distance from the zenith. Then in the triangle ZCM

$$\text{Rsin (arc ZM)} = \text{Sun's } \textit{madhyajyā},$$

$$\angle \text{ZCM} = 90°,$$

$$\text{and Rsin (MZC)} = \text{Sun's } \textit{udayajyā}.$$

Therefore

$$\text{Rsin (arc MC)} = \frac{\text{Rsin (arc ZM)} \times \text{Rsin (MZC)}}{\text{R}}$$

$$= \frac{\text{Sun's } \textit{madhyajyā} \times \text{Sun's } \textit{udayajya}}{\text{R}}$$

The final result, *viz*.

$$\text{Sun's } \textit{dṛkkṣepajyā} = \sqrt{(\text{Sun's } \textit{madhyajyā})^2 - (\text{Rsin MC})^2}$$

is obtained by treating the triangle formed by the Rsines of the sides of the triangle ZCM as a plane right-angled triangle (which assumption is however incorrect).

The Moon's *dṛkkṣepajyā* has been similarly obtained by taking the Moon's orbit in place of the ecliptic.

Brahmagupta has rightly criticised the above rule for being inaccurate.[1]

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1. See *BrSpSi*, xi. 29-30.

DṚGGATIJYĀS OF THE SUN AND THE MOON

दृक्-दृक्क्षेपकृतिविशेषितस्य मूलं स्वदृग्गतिः कुवशात् ।

34. (i) The square root of the difference between the squares of (i) the Rsine of the zenith distance (of the Sun or Moon) and (ii) the *dṛkkṣepajyā*, is the (Sun's or Moon's) own *dṛggatijyā*.[1]

The Sun's *dṛggatijyā* is the Rsine of the arcual distance of the zenith from the secondary to the ecliptic passing through the Sun.

The Moon's *dṛggatijyā* is the Rsine of the arcual distance of the zenith from the secondary to the Moon's orbit passing through the Moon.

The formula for the (Sun's or Moon's) *dṛggatijyā* stated in the text is

$$dṛggatijyā = \sqrt{[\text{Rsin (z.d.)}]^2 - (dṛkkṣepajyā)^2}\,.$$

This formula is correct and can be proved as follows: Let CS be the ecliptic and K its pole; S the Sun and Z the zenith; KZC and KS the secondaries to the ecliptic; and ZA the perpendicular to KS. Since the arcs ZC and ZA are perpendicular to CS and AS respectively,

$$(\text{Rsin ZA})^2 = (\text{Rsin ZS})^2 - (\text{Rsin ZC})^2,$$

i.e., (Sun's *dṛggatijyā*)^2 = (Sun's *dṛgjyā*)^2 — (Sun's *dṛkkṣepajyā*)^2.

Similarly,

(Moon's *dṛggatijyā*)^2 = (Moon's *dṛgjyā*)^2 — (Moon's *dṛkkṣepajyā*)^2.

According to Brahmagupta (A.D. 628), this is wrong. Says he: "*Dṛkkṣepajyā* is the base and *dṛgjyā* the hypotenuse; the square root of the difference between their squares is the *dṛṅnatijya* (=*dṛggatijyā*). This configuration is also improper."[2]

Brahmagupta's criticism is valid if the *dṛggati* means "the arc of the ecliptic lying between the central ecliptic point and the Sun or Moon" as explained by the commentator Sūryadeva.

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1. *Cf. MBh*, v. 23; *LBh*, v, 7(c-d)-8 (a-b).
2. *BrSpSi*, xi. 27.

PARALLAX OF THE SUN AND THE MOON

कुवशात् ।
क्षितिजे स्वा दृक्छाया[1] भूव्यासार्धं नभोमध्यात् ॥ ३४ ॥

34. (ii) On account of (the sphericity of) the Earth, parallex increases from zero at the zenith to the maximum value equal to the Earth's semi-diameter (as measured in the spheres of the Sun and the Moon) at the horizon.

The word *dṛkchāyā* in the text means parallax.

The instruction of the text implies, according to the commentators, the following formulae :

$$\text{parallax in longitude} = \frac{\text{Earth's semi-diameter} \times \textit{dṛggatijyā}}{\text{R}} \ \textit{yojanas}$$

in the sphere of the planet concerned

$$= \frac{\text{Earth's semi-diameter} \times \textit{dṛggatijyā}}{\text{planet's true distance in } \textit{yojanas}} \ \text{minutes.}$$

$$\text{parallax in latitude} = \frac{\text{Earth's semi-diameter} \times \textit{dṛkkṣepajyā}}{\text{R}} \ \textit{yojanas}$$

$$= \frac{\text{Earth's semi-diameter} \times \textit{dṛkkṣepajyā}}{\text{planet's true distance in } \textit{yojanas}} \ \text{minutes.}$$

On the use of the word *svadṛkkṣepa*, Bhāskara I observes :

> "The orbits of the Sun and the Moon being different, the (five) Rsines (*viz., udayajyā, madhyajyā, dṛkkṣepajyā, dṛgjyā* and *dṛggatijyā*) for them are said to differ. This difference is indicated by the words '*svadṛkkṣepa*' etc. of the Master (Āryabhaṭa I)."[2]

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1. E. स्वसुदृक्छाया

2. *MBh*, v. 12,

2. *The visibility corrections*

VISIBILITY CORRECTION *AKṢADṚKKARMA* FOR THE MOON

विक्षेपगुणाक्षज्या लम्बकभक्ता[1] भवेद् ऋणमुदक्स्थे[2] ।
उदये धनमस्तमये दक्षिणगे धनमृणं चन्द्रे ॥ ३५ ॥

35. **Multiply the Rsine of the latitude of the local place by the Moon's latitude and divide (the resulting product) by the Rsine of the colatitude : (the result is the *akṣadṛkkarma*) for the Moon). When the Moon is to the north (of the ecliptic), it should be subtracted from the Moon's longitude in the case of the rising of the Moon and added to the Moon's longitude in the case of the setting of the Moon ; when the Moon is to the south (of the ecliptic), it should be added to the Moon's longitude (in the case of the rising of the Moon) and subtracted from the Moon's longitude (in the the case of the setting of the Moon).**[3]
That is

$$akṣadṛkkarma = \frac{\text{Rsin } \phi \times \text{Moon's latitude}}{\text{Rcos } \phi}.$$

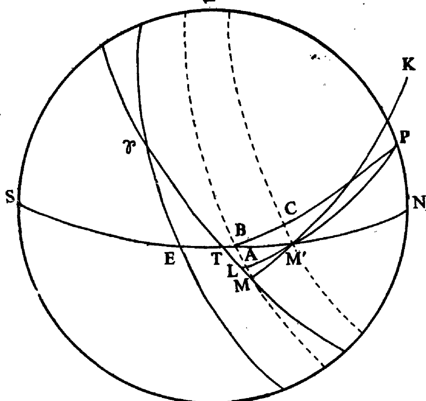


Fig. 11

1. A.B.C.E.F. Nī. Ra. Sū. भजिता; D. विहृता 2. D. मुदक्स्थेऽर्के

3. The same rule occurs in *PSi* (Pauliśa), v. 8 ; *KK*, I, vi. 3 ; *MBh*, vi. 1-2(a-b) ; *LBh*, vi. 1-2 ; *KR*, v. 2,

Let the figure represent the Celestial Sphere (*Khagoḷa*) for the local place in latitude ϕ, SEN is the eastern horizon and Z the zenith : γE is the equator and P its north pole ; γT is the ecliptic and K its north pole. Suppose that the Moon is rising at the point M′ on the horizon. Let M be the point where the secondary to the ecliptic drawn through M′ meets the ecliptic, L the point where the hour circle through M′ meets the ecliptic and T the point where the horizon intersects the ecliptic. Then the arc TL of the ecliptic is called the *akṣadṛkkarma* and the arc LM of the ecliptic is called the *ayanadṛkkarma.*

Let A be the point where the diurnal circle through M intersects the hour circle through M′ and B the point where the diurnal circle through M intersects the horizon. Then, since MM′ is small, regarding the triangle M′AB as plane, we have

$$\text{arc AB} = \frac{\text{Rsin (BM}'\text{A)} \times \text{M}'\text{A}}{\text{Rsin (M}'\text{BA)}} \text{ approx.}$$

$$= \frac{\text{Rsin (BM}'\text{A)} \times \text{M}'\text{M}}{\text{Rsin (M}'\text{BA)}}$$

$$= \frac{\text{Rsin } \phi \times \text{Moon's latitude}}{\text{Rcos } \phi},$$

Assuming the *akṣadṛkkarma* as roughly equal to arc AB, Āryabhaṭa gives

$$akṣadṛkkarma = \frac{\text{Rsin } \phi \times \text{Moon's latitude}}{\text{Rcos } \phi}$$

This rule is generally used when the celestial latitude of the body concerned is small. When the celestial latitude is large, a more accurate rule is prescribed.[1]

VISIBILITY CORRECTION *AYANADṚKKARMA*
OF THE MOON

विक्षेपापक्रमगुणमुत्क्रमणं विस्तरार्धकृतिभक्तम् ।
उदगृणधनमुदगयने दक्षिणगे धनमृणं याम्ये ॥ ३६ ॥

36. Multiply the Rversed sine of the Moon's (tropical) longitude (as increased by three signs) by the Moon's latitude and also by the (Rsine of the Sun's) greatest declination and divide (the resulting

1. See *BrSpSi*, x. 18-19 ; *ŚiDVṛ*, *I*, xi. 12-13 ; and *SiŚi I*, vii. 6. Bhāskara II gives a slightly modified formula for small celestial latitude also. See *SiŚi*, *I*, vii, 7. The most accurate formula occurs in *SiTV*, vii. 103-104.

product) by the square of the radius. When the Moon's latitude is north, it should be subtracted from or added to the Moon's longitude, according as the Moon's *ayana* is north or south (*i.e.*, according as the Moon is in the six signs beginning with the tropical sign Capricorn or in those beginning with the tropical sign Cancer) ; when the Moon's latitude is south, it should be added or subtracted, (respectively).[1]

That is

$$ayanadṛkkarma = \frac{\text{Rvers }(M+90°) \times \text{Moon's latitude} \times \text{Rsin } 24°}{R^2}$$

where M is the Moon's tropical longitude.

The rationale of the formula is as follows :

From triangle M′MA (See Fig. 11, p. 148), we have

$$\text{arc MA} = \frac{\text{Rsin (MM'A)} \times \text{Rsin (arc MM')}}{R} \text{ approx.}$$

$$= \frac{ayanavalana \times \text{Moon's latitude}}{R} \text{ approx.}$$

But (vide infra vs. 45), we have

$$ayanavalana = \frac{\text{Rvers }(M+90°) \times \text{R sin } 24°}{R}$$

$$\therefore \text{arc MA} = \frac{\text{Rvers }(M+90°) \times \text{Moon's latitude} \times \text{Rsin } 24°}{R^2}$$

Assuming the arc LM of the ecliptic (which denotes the *ayanadṛkkarma*) as approximately equal to arc MA, we have

$$ayanadṛkkarma = \frac{\text{R vers }(M+90°) \times \text{Moon's latitude} \times \text{Rsin } 24°}{R^2}$$

When the *ayanadṛkkarma* and *akṣadṛkkarma* are applied to the rising or setting Moon, we get the longitude of that point of the ecliptic which rises or sets with the Moon.

There is difference of opinion regarding the interpretation of the word *utkramaṇam*. The commentator Someśvara interprets it as meaning "The Rversed sine of the Moon's longitude as increased by three signs", whereas the commentators Bhāskara I, Sūryadeva and Parameśvara interpret it as meaning "The Rversed sine of the

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1. The same rule occurs in *KK, I,* vi. 2 ; *MBh,* vi. 2 (c-d)-3 ; *LBh,* vi. 3-4 ; *KR,* v. 3. More accurate formulae occur in *SiŚi,* I, vii. 4, 5 and in *ŚiTV,* vii. 77-80.

Moon's longitude as diminished by three signs."[1] The commentator Raghunātha-rāja interprets it as meaning Rvers (M+90°) or Rvers (M−90°), according as the desired *ayana* commences with Capricorn or with Cancer.

We have followed Someśvara's interpetation, because it agrees with the teachings of Āryabhaṭa in stanza 45 below and also because it agrees with the teachings in his midnight system.[2]

Brahmagupta has modified this rule by replacing the Rversed sine of the Moon's longitude as increased by three signs by the Rsine of the same. The commentator Nīlakaṇṭha, however, interprets the word *utkramaṇam* itself as meaning "the Rsine of the complement of the Moon's longitude".

(4) Eclipses of the Moon and the Sun

CONSTITUTION OF THE MOON, SUN, EARTH AND SHADOW AND THE ECLIPSERS OF THE SUN AND MOON

चन्द्रो जलमर्कोऽग्निः [3]मृद्भूश्च्छायापि या तमस्तद्धि ।
छादयति शशी सूर्यं, शशिनं महती च भूच्छाया ॥ ३७ ॥

37. The Moon is water, the Sun is fire, the Earth is earth, and what is called Shadow is darkness (caused by the Earth's Shadow). The Moon eclipses the Sun and the great Shadow of the Earth eclipses the Moon.

The stat ement that the Moon is water has proved false.

OCCURRENCE OF AN ECLIPSE

स्फुटशशिमासान्तेऽर्कं पातासन्नो यदा प्रविशतीन्दुः ।
भूच्छायां पक्षान्ते तदाधिकोनं ग्रहणमध्यम् ॥ ३८ ॥

38. When at the end of a lunar month, the Moon, lying near a node (of the Moon), enters the Sun, or, at the end of a lunar fortnight, enters the Earth's Shadow, it is more or less the middle of an eclipse, (solar eclipse in the former case and lunar eclipse in the latter case).

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1. Govinda-svāmi, too, says the same thing. Writes he : क्व पुनरिहोत्क्रमज्या गृह्यते ? कोट्यामिति ब्रूमः । सर्वत्र हि कोट्या एवोत्क्रमणं न्याय्यम् । अत एव [भास्करेण] वर्जितत्रिभवनस्येत्युक्तम् । See his comm. on *MBh*, vi. 3.
2. See *KK*, I, vi. 2.
3. So. मृद्भूच्छायापि

Āryabhaṭa evidently takes the time of conjunction of the Sun and Moon as the middle of a solar eclipse, and the time of opposition of the Sun and Moon as the middle of a lunar eclipse. This is only approximately true.

The phrase "more or less", according to the commentators, is indicative of the fact that, on account of parallax, the time of apparent conjunction is not exactly the same as that of geocentric conjuction.

LENGTH OF THE SHADOW

भूरविविवरं विभजेद् भूगुणितं तु रविभूविशेषेण ।
भूच्छायादीर्घत्वं[1] लब्धं भूगोलविष्कम्भात् ॥ ३९ ॥

39. **Multiply the distance of the Sun from the Earth by the diameter of the Earth and divide (the product) by the difference between the diameters of the Sun and the Earth: the result is the length of the Shadow of the Earth (*i.e.* the distance of the vertex of the Earth's shadow) from the diameter of the Earth (*i.e.* from the centre of the Earth).**[2]

That is,

$$\text{length of Earth's Shadow} = \frac{\text{Sun's distance} \times \text{Earth's diameter}}{\text{Sun's diameter} - \text{Earth's diameter}}$$

The Hindu method for deriving this formula, called "The lamp and Shadow method" (*pradīpacchāyā-karma*), is as follows :

Consider the figure below. S is the centre of the sun and E that of the Earth. SA and EC are drawn perpendicular to SE and denote the semi-diameters of the Sun and the Earth, respectively. BC is parallel to SE. V is the point where SE and AC produced meet each other.

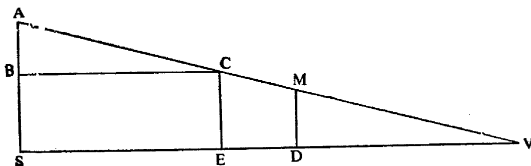


Fig. 12

1. C. E. Nī. Sū. छायाया दीर्घत्वं
2. *Cf. BrSpSi*, xxiii. 8 ; *MBh*, v. 71 ; *LBh*, iv. 6.

Hindu astronomers compare SA with a lamp post, EC with a gnomon, and EV with the shadow cast by the gnomon due to the light of the lamp. Consequently, they call EV 'the length of the Earth's shadow from the diameter of the Earth'.

The triangles CEV and ABC are similar ; therefore

$$\frac{EV}{EC}=\frac{BC}{AB}=\frac{SE}{SA-EC}$$

$$\therefore \quad EV=\frac{SE\times EC}{SA-EC}=\frac{SE\times 2\,EC}{2\,SA-2\,EC}.$$

i.e., length of Earth's shadow $=\dfrac{\text{Sun's distance}\times\text{Earth's diameter}}{\text{Sun's diameter}-\text{Earth's diameter}}$.

EARTH'S SHADOW AT THE MOON'S DISTANCE

छायाग्रचन्द्रविवरं भूविष्कम्भेण तत् समभ्यस्तम्[1] ।
भूच्छायया विभक्तं विद्यात् तमसः स्वविष्कम्भम्[2] ॥ ४० ॥

40. Multiply the difference between the length of the Earth's shadow and the distance of the Moon by the Earth's diameter and divide (the product) by the length of the Earth's shadow : the result is the diameter of the *Tamas* (*i. e.*, the diameter of the Earth's shadow at the Moon's distance).[3]

That is

Diameter of *Tamas*

$$=\frac{(\text{length of Earth's shadow}-\text{Moon's distance})\times\text{Earth's diameter}}{\text{length of Earth's shadow}}.$$

The rationale of this formula is as follows :

See the previous figure (Fig. 12). M is the position of the Moon when it is just on the point of entering the Earth's Shadow. MD is perpendicular to SE ; so it denotes the semi-diameter of the Earth's shadow at the Moon's distance, *i.e.*, the semi-diameter of the *Tamas*.

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1. B. D. समभ्यस्य
2. Pa. स्वविष्कम्भ:
3. *Cf. BrSpSi*, xxiii. 8-9 ; *MBh*, v. 72 ; *LBh*, iv. 7.

The triangles MDV and CEV are similar ; therefore

$$\frac{MD}{DV}=\frac{CE}{EV}$$

$$\text{or} \quad MD=\frac{(EV-ED)\times CE}{EV}$$

$$\text{or} \quad 2\,MD=\frac{(EV-ED)\times 2\,CE}{EV}=\frac{(EV-EM)\times 2\,CE}{EV} \text{ approx.}$$

i.e., Diameter of *Tamas*

$$=\frac{(\text{length of Earth's shadow}-\text{Moon's distance})\times\text{Earth's dameter}}{\text{length of Earth's shadow}}.$$

approx.

HALF-DURATION OF A LUNAR ECLIPSE

तच्छशिसम्पर्कार्धकृतेः[1] शशिविक्षेपवर्गितं[2] शोध्यम्[3] ।
स्थित्यर्धमस्य मूलं[4] ज्ञेयं चन्द्रार्कदिनभोगात् ॥ ४१ ॥

41. **From the square of half the sum of the diameters of that (*Tamas*) and the Moon, subtract the square of the Moon's latitude, and (then) take the square root of the difference : the result is known as half the duration of the eclipse (in terms of minutes of arc). The corresponding time (in *ghaṭīs* etc.) is obtained with the help of the daily motions of the Sun and the Moon.[5]**

That is,

Half the duration of a lunar eclipse

$$=\sqrt{\sigma^2-\beta^2} \text{ minutes of arc}$$

$$=\frac{60\times\sqrt{\sigma^2-\beta^2}}{d} \text{ } ghaṭīs$$

where σ = sum of the semi-diameters of *Tamas* and Moon

β = Moon's latitude

d = (Moon's daily motion − Sun's daily motion) in minutes of arc.

---

1. F. Pa. सम्पर्कार्धस्य कृतेः
2. Gh. So. विक्षेपवर्गितं ; others विक्षेपस्य वर्गितं
3. A. B. D. Nī. वर्गितमपोह्य
4. A. D. Nī. स्थित्यर्धं तन्मूलं
5. *Cf. MBh*, v. 74-76 (a-b) ; *LBh*, iv. 10-12 ; *KK*, I, iv. 4.

This gives only an approximate value of the semi-duration of the eclipse. To obtain the best approximation, the process should be iterated until the value of the semi-duration is fixed. For details, see *MBh*, v. 75-76.

HALF-DURATION OF TOTALITY OF A LUNAR ECLIPSE

चन्द्रव्यासार्धोनस्य वर्गितं यत्तमोमयार्धस्य ।
विक्षेपकृतिविहीनं तस्मान्मूलं विमर्दार्धम् ॥ ४२ ॥[1]

42. Subtract the semi-diameter of the Moon from the semi-diameter of that *Tamas* and find the square of that difference. Diminish that by the square of the (Moon's) latitude and then take the square root of that : the square root (thus obtained) is half the duration of totality of the eclipse.[2]

That is,

half the duration of totality $= \sqrt{s^2 - \beta^2}$ minutes of arc

$$= \frac{60 \times \sqrt{s^2 - \beta^2}}{d} \ ghaṭīs,$$

where s = semi-diameter of *Tamas* — semi-diameter of Moon

β = Moon's latitude

d = (Moon's daily motion—Sun's daily motion) in minutes of arc.

This also gives only a rough approximation. To obtain the best approximation, the process should be iterated until the semi-duration of totality is fixed.

THE PART OF THE MOON NOT ECLIPSED

तमसो विष्कम्भार्धं शशिविष्कम्भार्धवर्जितमपोह्य ।
विक्षेपाद्यच्छेषं न गृह्यते तच्छशाङ्कस्य[3] ॥ ४३ ॥

43. Subtract the Moon's semi-diameter from the semi-diameter of the *Tamas*; then subtract whatever is obtained from the

1. F. transposes this verse to after 44.

2. *Cf. PSi*, x. 7 ; *MBh*, v. 76(c-d) ; *LBh*, iv. 14 ; *KK*, *I*, iv. 4.

3. Nī. The text in the TSS edn. adds भूच्छाया, which is not warranted by the Com. of Nī. Moreover, it is metrically superfluous.

Moon's latitude : the result is the part of the Moon not eclipsed (by the *Tamas*).[1]

That is,

the length of the Moon's diameter which is not eclipsed

=Moon's latitude—(semi-diameter of *Tamas*—Moon's semi-diameter).

It is easy to see that the obscured part of the Moon's diameter (at the time of opposition of the Sun and Moon in a partial lunar eclipse)

$=\frac{1}{2}$ (diameter of *Tamas*+diameter of Moon)—Moon's latitude, and hence the unobscured part of the Moon's diameter at that time

=Moon's diameter—{$\frac{1}{2}$(diameter of *Tamas*+diameter of Moon) —Moon's latitude}

=Moon's latitude—(semi-diameter of *Tamas*—semi-diameter of Moon).

As stated earlier, Āryabhaṭa I does not make any distinction between the time of opposition and the time of the middle of the eclipse. Hence the above rule.

MEASURE OF THE ECLIPSE AT THE GIVEN TIME

विक्षेपवर्गसहितात् स्थितिमध्यादिष्टवर्जितान्मूलम् ।
सम्पर्कार्धाच्छोध्यं शेषस्तात्कालिको ग्रासः ॥ ४४ ॥

44. **Subtract the *iṣṭa* from the semi-duration of the eclipse ; to (the square of) that (difference) add the square of the Moon's latitude (at the given time) ; and take the square root of this sum. Subtract that (square root) from the sum of the semi-diameters of the *Tamas* and the Moon : the remainder (thus obtained) is the measure of the eclipse at the given time.**

The term *iṣṭa* denotes, says the commentator Sūryadeva, the Moon's motion (in longitude) relative to *Tamas* corresponding to the time elapsed at the given time, since the first contact, or to elapse at the given time before the last contact.

Let AB be the ecliptic, the circle centred at T the *Tamas* at the time of opposition of the Sun and Moon, CD the Moon's orbit

---

1. The same rule occurs in *LBh*, iv. 9.
2. A. B. C. D. F. Ni. Pa. Su. स्थित्यर्धादिष्ट

relative to the *Tamas* at T. M_1 the position of the Moon at the time of the first contact, and M_2 the position of the Moon at the given time. M_1P and M_2Q are perpendiculars dropped on AB. Then

PT denotes the semi-duration of the eclipse,

PQ denotes the *iṣṭa*,

M_2Q denotes the Moon's latitude at the given time,

and LM denotes the eclipsed portion of the Moon's diameter at the given time.

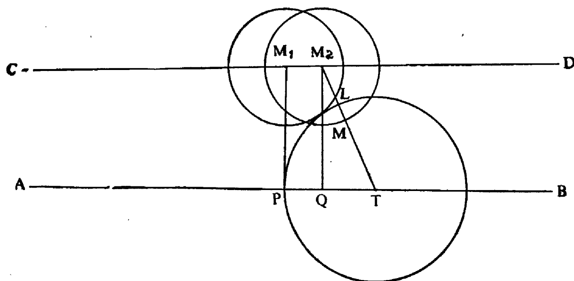


Fig. 13

It is evident from the figure that

$$LM=(LT+M_2M)-M_2T,$$

where

$$M_2T=\sqrt{M_2Q^2+QT^2}$$

$$=\sqrt{M_2Q^2+(PT-PQ)^2}.$$

Hence the above.

The *iṣṭa* is generally given in terms of time (in *ghaṭīs*) elapsed since the first contact or to elapse before the last contact, and the above rule is stated as follows :

"Subtract the *iṣṭa* (*ghaṭīs*) from the *ghaṭīs* of the semi-duration of the eclipse. Multiply that by the difference between the true daily motions of the Sun and Moon and divide by 60. Add the square of that to the square of the Moon's latitude for the given time, and take the square root (of that sum). This subtracted from the sum of the semi-diameters of the *Tamas* and the Moon gives the measure of the Moon's diameter eclipsed at the given time."

The portion *sthitimadhyādiṣṭavarjitānmūlam* of the text is defective as it does not convey the correct sense intended here. A correct reading would have been *viṣṭasthityardhavargitānmūlam.*

AKṢAVALANA

मध्याह्नोत्क्रमगुणितोऽक्षो दक्षिणतोऽर्धविस्तरहृतो दिक् ।

45. (a-b) **Multiply the Rversed sine of the hour angle (east or west) by (the Rsine of) the latitude, and divide by the radius : the result is the *akṣavalana*. Its direction (towards the east of the body in the afternoon and towards the west of the body in the forenoon) is south. (In the contrary case, it is north).**[1]

That is,

$$akṣavalana = \frac{\text{Rvers H} \times \text{Rsin } \phi}{\text{R}},$$

where H is the hour angle of the eclipsed body and ϕ the latitude of local place.

The *akṣavalana* is the deflection of the equator from the prime vertical on the horizon of the eclipsed body.

The above formula is incorrect. Brahmagupta (A.D. 628) modified it by replacing Rvers H by Rsin H.[2] Better and accurate formulae were given by Bhaskara II (A.D. 1150).[3]

The word *dik* in the text means *valana*.

The *Pauliśa-siddhānta* summarised by Varāhamihira gives the following rule : [4]

$$akṣavalana = \frac{\text{H} \times \phi}{90}$$

---

1. The same rule occurs in *PSi*, xi. 2 ; *MBh*, v. 42-44 ; *LBh*, iv. 15-16 ; *KK*, I, iv. 7 (as interpreted by Bhaṭṭotpala) ; *SMT*, lunar eclipse.

2. See *BrSpSi*, iv. 16.

3. See *SiŚi*, *I*, v. 20-21(a-b) ; *II*, viii. 68 ; and *II*, viii. 66(c-d)-67.

4. See *PSi*, vi. 8.

The formula of the old *Sūrya-siddhānta* summarised by Varāhamihira is :[1]

$$akṣavalana = \frac{\text{Rsin H} \times \text{R sin } \phi}{\text{R}},$$

which is the same as given by Brahmagupta.

AYANAVALANA FOR THE FIRST CONTACT

स्थित्यर्धाच्चार्केन्द्वोस्त्रिराशिसहितायनात् स्पर्शे ॥ ४५ ॥[2]

45. (c-d) **Making use of the semi-duration of the eclipse, calculate the longitude of the Sun or Moon (whichever is eclipsed) for the time of the first contact. Increase that longitude by three sign[3] and (multiplying the Rversed sine thereof by the Rsine of the Sun's greatest declination and dividing by the radius) calculate the Rsine of the corresponding declination : this is the *ayanavalana* (or *krāntivalana*) for the time of the first contact.**

(Its direction in the eastern side of the eclipsed body is the same as that of the *ayana* of the eclipsed body ; in the western side it is contrary to that).[4]

That is,

$$ayanavalana = \frac{\text{Rvers } (M + 90°) \times \text{Rsin } 24°}{\text{R}},$$

where M is the longitude of the eclipsed body, the Sun or Moon.

---

1. See *PSi*, xi. 2.

2. A. E. transpose this verse to after 46.

3. Govindasvāmi writes : त्रिराशिसहितेति शब्देनेह कोटिरभिधीयते । अयनशब्देन चापक्रमः । उत्क्रमशब्दश्चानुवर्तते । तेन कोट्युत्क्रमज्यापमगुण इत्यर्थः । So also writes Parameśvara : त्रिराशिसहितस्य भुजोत्क्रमज्याग्रहणो कोट्युत्क्रमज्याग्रहणं भवतीत्यर्थः । See Govindasvami's comm. on *MBh*, v. 46-47 and Parameśvara's supercommentary *Sidhānta-dīpika* on. vs. 45.

4. The same rule occurs in *PSi*, xi. 3 ; *MBh*, v. 45 ; *KK*, I, iv. 7. Also see *LBh*, iv. 17 and *SMT*, ch. on lunar eclipse, where $M + 90°$ is replaced by $M - 90°$. *KR*, iii. 14(c-d)-15(a-b) gives a mixed rule.

The *ayanavalana* is the deflection of the ecliptic from the equator on the horizon of the eclipsed body. It is defined as above for the first contact because in the case of a lunar eclipse in the eastern side (which is first eclipsed) the direction of the *ayanavalana* is the same as that of the Moon's *ayana*. The first contact is only a token; for the middle of the eclipse or for the last contact, it is obtained similarly.

The above formula for the *ayanavalana* is incorrect. It was modified by Brahmagupta, who replaced Rvers (M+90°) in the formula by Rsin (M+90°).[1] An accurate expression for the *ayanavalana* was given by Bhāskara II (1150).[2]

The *akṣavalana* the *ayanavalana*, and the *vikṣepavalana* are required in the graphic representation of an eclipse. For the details of graphic representation of an eclipse according to Bhāskara I, the reader is referred to *MBh*, v. 46-47 and *LBh*, iv. 19-32.

The commentators of the *Āryabhaṭīya* are of the opinion that the word *sthityardhācca* refers to the *vikṣepavalana*. "Since the Moon's latitude is obtained from the *sthityardha*", writes the commentator Someśvara, "the Moon's latitude is meant by the word *sthityardha*". So also says the commentator Parameśvara: "By the word *sthityardha* are meant the moon's latitude corrected for parallax (in the case of a solar eclipse) and the Moon's latitude (in the case of a lunar eclipse) which are based on that".

Thus, according to the interpretation of the commentators, the verse 45 (c-d) should be translated as follows:

> **"With the help of the semi-duration of the eclipse, calculate (the Moon's latitude for the first contact, reverse its direction in the case of a lunar eclipse, and treat it as the *vikṣepa*)*valana*.**
>
> **Also calculate the longitude of the Sun or Moon (whichever is eclipsed) for the time of the first contact and increase it by three signs and (multiplying the Rversed sine thereof by the Rsine of the Sun's greatest decli-**

1. See *BrSpSi*, iv. 17.
2. See *SiŚi*, I, v. 21(c-d)-22(a-b).

nation and dividing by the radius) calculate the Rsine of the corresponding declination : this is the *ayanavalana* (or *krāntivalana*) for the time of the first contact.

(Its direction in the eastern half of the eclipsed body is the same as that of the *ayana* of the eclipsed body ; in the western side it is contrary to that.)"

The word *sparśe* in the text means 'at the time of the first contact', which seems to suggest that the calculation is to be made for the first contact. But this is not the case. "The time of the first contact is only a token", writes the commentator Sūryadeva. "The calculation of the *valana* should be made for the time of the first contact, the time of last contact, the time of the middle of the eclipse, and for any other desired time."

COLOUR OF THE MOON DURING ECLIPSE

प्रग्रहणान्ते धूम्रः खण्डग्रहणे शशी भवति कृष्णः ।
सर्वग्रासे कपिलः सकृष्णताम्रस्तमोमध्ये ॥ ४६ ॥

46. **At the beginning and end of its eclipse, the Moon (*i.e.*, the obscured part of the Moon) is smoky ; when half obscured, it is black ; when (just) totally obscured, (*i e.*, at immersion or emersion), it is tawny ; when far inside the Shadow, it is copper-coloured with blackish tinge.**[1]

In the case of a solar eclipse, the obscured part of the Sun looks black at every phase of the eclipse.

WHEN THE SUN'S ECLIPSE IS NOT TO BE PREDICTED

सूर्येन्दु[2]परिधियोगेऽर्काष्टमभागो भवत्यनादेश्यः ।
भानोर्भासुरभावात्[3] स्वच्छतनुत्वाच्च[4] शशिपरिधेः ॥ ४७ ॥

47. **When the discs of the Sun and the Moon come into contact, a solar eclipse should not be predicted when it amounts to**

1. *Cf. PSi* (Pauliśa), vi. 9(*c-d*)-10.
2. Pa. gives a variant, अर्केन्दु
3. So. भास्वरभावात्
4. Ni. स्वच्छतमत्वाच्च

one-eighth of the Sun's diameter (or less) (as it may not be visible to the naked eye) on account of the brilliancy of the Sun and the transparency of the Moon.[1]

PLANETS DETERMINED FROM OBSERVATION

क्षितिरवियोगाद् दिनकृद् रवीन्दुयोगात् प्रसाधितश्चेन्दुः[2] ।
शशिताराग्रहयोगात् तथैव ताराग्रहाः सर्वे ॥ ४८ ॥

48. **The Sun has been determined from the conjuction of the Earth and the Sun, the Moon from the conjunction of the Sun and the Moon, and all the other planets from the conjunctions of the planets and the Moon.**

What is meant is that the revolution numbers etc. of the Sun, Moon and the planets stated in the first chapter of the present work have been determined by observing the conjunctions as stated above.

According to the commentator Someśvara, the method of finding the revolutions of the Sun, Moon and the planets in a *yuga* by observing the conjunctions is as follows :

(1) *Revolutions of the Sun*

Let the number of conjunctions of the Sun and the Earth, *i.e.*, the number of civil days (including the fraction of a civil day) in one (sidereal) solar year be c. Then the number of revolutions of the Sun in a *yuga* $=C/c$, where C denotes the number of civil days in a *yuga*.

(2) *Revolutions of the Moon*

Let the number of conjunctions (including the fraction of a conjunction) of the Sun and the moon in one solar year be c. Then the number of revolutions of the Moon in a *yuga* $=(c+1)S$, where S denotes the number of solar years in a *yuga*.

---

1. *Cf. MBh,* v. 41.
2. Ra. प्रसाधितश्चंद्रः; So. प्रसाधयेत् चन्द्रम्

Alternative method. Let the number of risings of the Moon (including the fraction also) in one solar year be m. Then the number of revolutions of the Moon in a *yuga*=E−mS, where E denotes the number of rotations of the Earth and S the number of solar years in a *yuga*.

(3) *Revolutions of the planets (Mars, Mercury, Jupiter, Venus and Saturn) round the Earth*

Let the number of conjunctions of a planet and the Moon (including the fraction also) in one solar year be c. Then the number of revolutions of the planet in a *yuga*=M−cS, where M denotes the number of revolutions of the Moon and S the number of solar years in a *yuga*.

(4) *Revolutions of the śīghroccas of Mercury and Venus*

Let C denote the number of civil days in a *yuga*, and d the number of days (including the fraction of a day) between two consecutive inferior or superior conjunctions of the planet and the Sun. Then

C/d=revolution-number of the planet's *śīghrocca*− revolution-number of the Sun.

∴ number of revolutions of the planet's *śīghrocca* in a *yuga*

=C/d+Sun's revolutions in a *yuga*.

For other methods see Someśvara's Commentary, in Part II.

ACKNOWLEDGEMENT TO BRAHMĀ

सदसज्ज्ञानसमुद्रात् समुद्धृतं ब्रह्मणः[1] प्रसादेन ।
सज्ज्ञानोत्तमरत्नं मया निमग्नं स्वमतिनावा ॥ ४९ ॥

49. By the grace of Brahmā, the precious jewel of excellent knowledge (of astronomy) has been brought out by me by means of the boat of my intellect from the sea of true and false knowledge by diving deep into it.

1. All others except Bh. and So. read देवता for ब्रह्मणः

आर्यभटीयं नाम्ना पूर्वं स्वायम्भुवं सदा[1] नित्यम्[2] ।
सुकृतायुषोः[3] प्रणाशं कुरुते प्रतिकञ्चुकं योऽस्य ॥ ५० ॥

50. **This work, *Āryabhaṭīya* by name, is the same as the ancient *Svāyambhuva* (which was revealed by Syayambhū) and as such it is true for all times. One who imitates it or finds fault with it shall lose his good deeds and longevity.**

[इति गोलपादः समाप्तः]

इत्यार्यभटाचार्यकृतम् आर्यभटीयं समाप्तम् ॥[4]

1. B. सदसि for सदा

2. Nī. सत्यम् ; Pa. सद्यत् ; Ra. So. नित्यम्

3. C. सुकृतायुषे ; F. सुकृतायुषः

4. A. इति गोलपादः । आर्यभटीयं समाप्तम् । 'सेव्यो दुग्धाब्धितल्पः' एषुतिक्कूटियन्नत्ते अहर्गणम् (in Mal., meaning 16,99,817 is the Kali day of the completion of the transcription). B. इत्यार्यभटीयं नाम गणितकालक्रियागोल-प्रबन्धः । C. No formal col. D. इत्यार्यभटाचार्यविरचितो गोलपादः समाप्तः । E. गोलपादः समाप्तः । इत्यार्यभटीयं परिपूर्णं यथावत् । F. इति गोलपादः । आर्यभटीयं समाप्तम् ।

APPENDIX I

INDEX OF HALF-VERSES AND KEY PASSAGES

APPENDIX II

INDEX-GLOSSARY OF TECHNICAL TERMS

USED BY ĀRYABHAṬA

[*Note* : In each reference, the initial figures 1, 2, 3 and 4 refer, respectively, to the *Gītikā*, *Gaṇita*, *Kālakriyā* and *Gola pādas* and the further figures to verse numbers in the respective *pādas*.]

अंश (=भाग) (1. degree) 1. 6 ; 3.14 ; (2. part), 4.14

अक्ष (1. latitude), 4.19; (2. Latitude-triangle), 4.23

अक्षज्या (Rsine latitude), 4.26, 35

अग्र (1. tip or end), 4.19; (2. residue or remainder), 2.32, 33

अग्रा (amplitude at rising or the Rsine thereof), 4.30

अग्रान्तर (residue-difference), 4.32

अघन (non-cube), 2.5

अङ्गुल (unit of length, 1/24th of a cubit), 1.8

अज (sign of Aries), 4.25

अधिकाग्र (greater remainder) 2.32, 33

अधिकाग्रभागहार (divisor corresponding to greater remainder), 4.32, 33

अधिमासक (=अधिमास), (intercalary month), 3.6

अनादेश्यग्रहण (eclipse not to be predicted), 4.47

अनुलोम (direct or anticlockwise), 3.17, 20, 21

अनुलोमग (having direct motion), 3.21

अनुलोमगति (having direct motion), 3.20 ; 4.9

अनुलोमविवर (distance between two planets in direct motion), 2.31

अन्तर (difference between two quantities), 2.24

अन्त्यपद (last term in a series), 2.19

अपक्रम (1. greatest declination), 1.8 ; (2. declination), 4.24, 26

अपचय (subtractive), 2.28

अपमण्डल (=अपक्रममण्डल) (ecliptic), 1.8 ; 4.1, 2, 3

अपसव्य (anticlockwise), 4.16

अपसर्पिणी (a designation of the second half of the *yuga*), 3.9

अभ्यास (multiplication), 2.9

अयन (northward or southward motion of a planet), 4.36

अर्काग्रा (Sun's amplitude at rising, or the Rsine thereof), 4.30

अर्कोदय (sunrise), 1.4

अर्धज्या (=ज्या), (Rsine), 2.17

अर्बुद, (=10^8), 2.2

अवर्ग (not belonging to the *varga-s* which as *ka-varga, ca-varga* etc.), 1. 2

अश्वयुक् (month of Āśvina), 3.4

अस्तमय (setting, diurnal or heliacal), 4.13

अस्तमयोदयसूत्र (rising-setting line), 4.29

अहोरात्रार्धविष्कम्भ (day-radius), 4.24, 25

आदि or आदिधन (first term), 2.19, 20

आयाम (length or breadth), 2.8

आयामक्षेत्र (Trapezium), 2.8

आर्क्ष (sidereal), 3.2

आसन्न (approximate), 2.10

इच्छा, see इच्छाराशि

इच्छाफल (fruit corresponding to *icchā*), 2.26

इच्छाराशि (requisition, one of the three quantities in the rule of three), 2.26

इन्दुपात (ascending node of the Moon), 4.2

इष्ट (desired or given number), 2.19

उच्च (*mandocca* or *śīghrocca*), 1.4 ; 3.4, 20

उच्चनीचपरिवर्त (anomalistic or synodic revolutions), 3.4

उच्चनीचवृत्त (epicycle), 3.19

उज्जयिनी (City of Ujjayinī, modern Ujjain in Madhya Bharat), 4.14

उत्क्रमण (उत्क्रमज्या) (Rversed sine), 4.36

उत्तर (common difference), 2.19, 20

उदगयन (उत्तरायण), (Sun's northward journey from winter solstice to summer solstice), 4.36

उत्सर्पिणी (a designation of the first half of the *Yuga*), 3. 9

उदयजीवा (उदयज्या) (Rsine amplitude of the rising point of the ecliptic), 4.33

उदयास्तमय (heliacal rising and setting), 4.10

उन्नतभाग (degrees of altitude), 4.32

उन्मण्डल (equatorial horizon), 4.19

उपचिति (a series in general), 2.21

ऊनाग्र-छेद or °भागहार (divisor corresponding to smaller remainder), 2.32, 33

ऊर्ध्वभुजा (altitude or vertical side), 2.6

ग्रह (planet), 1.13 ; 3.8, 17, 20, 21, 23, 25

ग्रहण (eclipse), 4.37 ff.

ग्रहणमध्य (middle of the eclipse), 4.38

ग्रास (1. measure of eclipse), 4.35 ; (2. erosion by overlapping), 2.18

घन (1. cube of a number), 2. 3 ; (2. solid cube), 2.5

घनगोल (solid sphere), 2.7

घनगोलफल (volume of a solid sphere), 2.7

घनचितिघन (sum of the series of cubes of natural numbers), 2.22

घनफल (volume), 2.7

घनभूमध्य (Earth's centre), 3.18

घनमूल (cube root), 2.5

चक्र (circle), 1.6 ; 4.2

चतुरश्र (quadrilateral), 2.3

चतुर्भुज (quadrilateral), 2.11, 13

चान्द्र (lunar), 3.6

चन्द्रोच्च (Moon's apogee), 1.4

चरदल (ascensional difference), 4.27

चान्द्रमास (lunar month), 3.6

चाप (arc), 2.11

चापज्यार्ध (=ज्या), 2.12

चिति (sum of a series of natural numbers), 2.21

चितिघन (sum of a series $\Sigma\Sigma n$), 2.21, 22

चितिवर्ग (square of the sum of a series of natural numbers), 2.22

छाया (shadow) ; 2. 14, 15, 16; 4.5, 37, 38, 39, 40

छायादीर्घत्व (length of the Earth's shadow), 4.39

छेद (denominator), 2.27

जीवा (Rsine), 4.23

जूक (sign Libra), 4.1

ज्या (Rsine), 1.12; 2.9

ज्यार्ध (=ज्या, Rsine), 1.12 ; 2.11, 12

तम (1. Section of Earth's shadow-cone at the Moon's distance), 4. 42 ; (2. Earth's Shadow), 4.37

तमोमय (section of Earth's shadow-cone at Moon's distance), 4.42

तमोविष्कम्भ (diameter of Shadow, *i.e.*, diameter of the Earth's shadow cone at Moon's distance), 4.40, 43

ताराग्रह (Star planets, *i. e.*, the planets Mars, Mercury, Jupiter, Venus and Saturn), 4.2

तिथि (lunar day), 3.6

तिथिप्रलय (=अवम, *i.e.*, omitted *tithi*), 3.6

तुला (sign Libra), 4.1

त्रिभुज (triangle), 2.6, 11, 13

परमापक्रमजीवा (°ज्या) (Rsine of the greatest declination), 4.30

परशङ्कु (परमशङ्कु) (Rsine of greatest altitude, *i.e.*, Rsine of meridian altitude), 4.32

परिणाह (periphery, circumference), 2.7, 10

परिधि (circumference), 2.9, 11

परिवर्त (revolution), 3.4

पात (ascending node), 2.4, 9; 4. 2, 3

पार्श्व (lateral or adjacent side), 2.8, 9

पितृवर्ष (year of the manes), 3.7

पूर्वापर (east-west), 4.31

प्रग्रहण (first contact in an eclipse), 4.46

प्रतिमण्डल (eccentric circle of a planet), 3.17, 18, 19

प्रतिलोम (retrograde, clockwise), 3.17, 20, 21

प्रमाण (argument in the rule of three), 2.26

प्रयुत (=10^6), 2.2

प्रवहवायु (provector wind), 4.10

प्राग्लग्न (=लग्न) (rising point of the ecliptic), 4.21

प्राग्ज्याकाष्ठ (right ascension), 4.25

प्राण (a unit of time equal to four sidereal seconds or one-sixth of a *vinaḍikā*), 1.6; 3.2

फल (interest on principal), 2.25

फलराशि (fruit, one of the three quantities in the rule of three), 2.26

बडवामुख (south pole), 4.12, 16

ब्रह्मदिवस (a day of Brahmā, a *Kalpa*), 3.8

भ (=नक्षत्र) (asterism), 1. 6; 3. 11; 4.9, 15

भगण (revolutions, number of revolutions performed by a planet in a *yuga*), 1. 3; 3.2, 3, 4

भगोल (sphere of asterisms, with its centre at the Earth's centre), 4.15, 16, 23

भपञ्जर (भचक्र), (circle of the asterisms), 4.10

भपरिणाह (circumference of the circle of asterisms), 3.12

भाग (degree), 4.32

भागहरण (division), 2.4

भागहार (भागहर), (divisor), 2. 27, 28, 32

भुजा (=भुज) (lateral side of a right angled triangle), 2.15, 16, 17

भूगोल (sphere of the Earth), 4.6, 7, 8

भूगोलविष्कम्भ (diameter of the Earth), 4.39, 40

भूच्छाया (Earth's shadow), 4. 38, 39, 40

लम्बक (1. plumb), 2. 13 ; (2. Rcosine of latitude), 4. 26, 29, 30, 35

वक्रि (a planet in retrograde motion), 1.1

वर्ग (1. square), 2.3, 4, 5, 14, 22 ; (2. odd places indicated by letters of the *vargas* *viz.*, *ka-varga*, *ca-varga* etc.), 1.2

वर्गचितिघन (sum of a series of squares of natural numbers), 2.22

वर्गमूल (square root), 2.4, 14

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विक्षेप (celestial latitude), 1. 8 ; 4.35, 42, 43, 44

विनाडिका (विघटिका), (one-sixtieth of a *ghaṭikā*), 3.1, 2

विमर्दार्ध (half-duration of the totality of an eclipse), 4.42

विलोम (retrograde), 1.4

विलोमविवर (difference of two planets, one direct and the other retrograde), 2.31

विवर (difference), 2.31

विशेष (difference), 2.15, 20; 4.33, 34, 39

विषुवज्जीवा (= अक्षज्या), (Rsine of latitude), 4.29, 31

विषुवत् (equator), 4.24

विष्कम्भ (diameter), 2. 7, 9, 11 ; 4.40

विष्कम्भार्ध (semi-diameter, radius), 2.7, 9, 11, 14

विस्तर (=विस्तार), (length), 2.8

वृत्त (1. circle), 2. 10, 11, 13, 17 ; (2. epicycle), 3.19, 20, 21

वृत्तपरिणाह (circumference of a circle), 2.10

वृत्तफल (area of a circle), 2.7

वृत्तपरिधि (circumference of a circle), 3.19

वृन्द (=10⁹), 2.2

वेग (velocity), 3.25

व्यतीपात, 3.3

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शशिदिवस (lunar day), 3.6

शशिमास (lunar month), 3.5

शीघ्रवृत्त (*śighra* epicycle), 1. 10, 11 ; 3.21

शीघ्रोच्च, (*śīghrocca*), 3.17, 22, 23, 24 ; 4.3

षडश्रि (a solid with six edges, a triangular pyramid), 2.6

APPENDIX III

SUBJECT INDEX

[*Note* :—The numbers refer to the relevant pages.]

1. Umasvāti is reputed to be one of the greatest metaphysicians of India and is held in high estimation by the two main sections of the Jainas. Unfortunately, his time and place of birth have not been settled definitely. According to the tradition of the Śvetāmbara Jainas, Umāsvāti was born in the now forgotten city of Nyagrodhikā. His name is said to have been a combination of the names of his parents, the father Svāti and the mother Umā. He was the disciple of the saint Ghoṣanandī. He lived about 150 B.C. His disciple Śyāmārya or Śyāmācārya, the author of the *Prajñāpanā-sūtra*, is said to have died 376 years after Śrī Vīra, that is, in 92 B.C. and his earliest (commentator is said to have been Siddhasena Gaṇi, or Divākara who lived *c.* 56 B.C. The Digambara tradition, on the other hand, sometimes, even changes his name and thinks it to be Umāsvāmī, not Umāsvāti. According to it, he lived from 135 A.D. to 219 A.D. Satish Chandra Vidyabhushan is of the opinion that he flourished in the first century A.D. All are, however, agreed on one point, *viz.*, that Umāsvāti resided in the city of Kusumapura (modern Patna).

APPENDIX IV

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